國立臺灣大學 106 學年度碩士班招生考試試題

第一大題選擇題：單選題與複選題之混合式試題，
考生應作答於『答案卡』（請勿作答於試卷之選擇題作答區）

1. (10%) Given an RLC circuit. The current i(t) can be expressed by $\frac{d^2i}{dt^2}+\frac{R}{L}\frac{di}{dt}+\frac{1}{LC}i=E(t)=0$ for $t>0$

where $R=1, C=\frac{2}{5}, L=\frac{1}{2}, i(0)=0, \frac{di(0)}{dt}=-8, \int_{-\infty}^{0} i(t)dt=\frac{8}{5}$

Which of the following are correct?

(A) i(t) approaches zero at $t\to\infty$ for any positive R, L ,C value,

(B) For t>0, i(t) does not have sinusoidal component,

(C) i(1) > e^{-2},

(D) For t>0, i(t) is proportional to e^{-t},

(E) i(π) = 0.

2. (5%) Solve the differential equation below.

$$y''+2y'+y=e^{-x}$$

Which of the following are the possible solutions?

(A) $y(x)=e^{-x}$,

(B) $y(x)=(5+\frac{1}{4}x^2)xe^{-x}$,

(C) $y(x)=e^{-x}+2xe^{-x}$,

(D) $y(x)=(2+\frac{1}{2}x)xe^{-x}$,

(E) $y(x)=e^{-x}+2x^2e^{-x}$.

3. (5%) For the differential equation below.

$$y^{(4)}+2y''+y=f(x)$$

Which of the following are correct?

(A) General solution form include $y(x)=x\cdot\cos(x)$,

(B) General solution form include $y(x)=e^{x}$,

(C) General solution form include $y(x)=\sin(x)$ and $y(x)=x\cdot\sin(x)$,

(D) If $f(x)=3e^{x}$, particular solution is in the form of c_1e^{x},

(E) If $f(x)=3\cos(x)$, particular solution is in the form of $c_1\cos(x)+c_2\sin(x)$.

見背面

4. (5%) A heart pacemaker consists of a switch, a battery, a capacitor with constant capacitance C, and the heart as a resister with constant resistance R. When the switch is closed, the capacitor charges; when the switch is open, the capacitor discharges, sending an electrical stimulus to the heart. During the time the heart is being stimulated, the voltage y(t) across the heart satisfies the linear differential equation :
$\frac{dy(t)}{dt}+\frac{1}{RC}y(t)=t$, where t is the time variable. Assume that initially, i.e., at t = 0, the voltage y(t) across the heart is equal to 4 and the constant $RC=1$. Which of the following statements are true?

(A) For $t\geq 0$, the particular solution of the linear differential equation is $y_p(t)=t-1$,

(B) For $t\geq 0$, the particular solution of the linear differential equation is $y_p(t)=4(t-1)$,

(C) For $t\geq 0$, the particular solution of the linear differential equation is $y_p(t)=ce^{-t}$, c is any constant,

(D) For $t\geq 0$, the solution of the linear differential equation is $y(t)=t-1+5e^{-t}$,

(E) For $t\geq 0$, the solution of the linear differential equation is $y(t)=4(t-1)+8e^{-t}$.

5. (5%) Consider one specially designed circuit with one capacitor, one inductor, and four resisters. The output voltage of the capacitor is denoted as x(t) and the output current of the inductor is denoted as y(t). Assume that initially x(0)=0 and y(0)=0 and these two devices might interact with each other by the following behavior. For the capacitor, the changing rate of x(t) is declined at a rate of -3x(t), and simultaneously increased at a rate of y(t), and also positively depends on the independent current source $g(t)=3t$. Similarly, for the inductor, the changing rate of y(t) declines at a rate of -4y(t), and simultaneously increases at a rate of 2x(t), and also positively depends on another independent voltage source $h(t)=e^{-t}$. Which of the following statements are true?

(A) $\frac{dx(t)}{dt}=-3x(t)+y(t)+3t$, (B) $\frac{dx(t)}{dt}=-3y(t)+x(t)+3t$, (C) $\frac{dx(t)}{dt}=-3x(t)+y(t)+e^{-t}$,

(D) $\frac{dy(t)}{dt}=2x(t)-4y(t)+e^{-t}$, (E) $\frac{dy(t)}{dt}=2x(t)-4y(t)+3t$.

6. (5%) A forced, undamped, and resonant motion of a mass on a spring can be described in the following equation: $\frac{d^2x(t)}{dt^2}+16x(t)=\cos(4t)$, where x(t) is the location of the mass. In the beginning, the mass is located at $x(0)=0$ and the initial velocity of the mass is equal to 1, i.e., $\frac{dx(0)}{dt}=1$. The Laplace transform X(s) of x(t) is:

接次頁

(A) $X(s)=\dfrac{2s+16}{(s^2+16)^2}$,　(B) $X(s)=\dfrac{s^2+s+16}{(s^2+16)^2}$,　(C) $X(s)=\dfrac{1}{s^2+16}+\dfrac{s}{(s^2+16)^2}$,

(D) $X(s)=\dfrac{1}{s+16}+\dfrac{s}{(s+16)^2}$,　(E) $X(s)=\dfrac{1}{s+16}+\dfrac{s}{s^2+16}$.

7. (5%) A semi-infinite plate coincides with the region defined by $0 \le x \le \pi, y \ge 0$. The left end is held at temperature: exp $(-y)$, and the right end is held at temperature zero for $y \ge 0$. The bottom of the plate is insulated.

$$u(x,y)=\frac{2}{\pi}\int_0^{\infty}\frac{\sinh\,[f(x)]}{g(\alpha)\sinh\,(\alpha\pi)}p(y)d\alpha$$

(A) $g(\alpha)=1+\alpha$,　(B) $f(x)=\alpha x$,　(C) $f(x)=\alpha(\pi-x)$,　(D) $p(y)=\sin(\alpha y)$,
(E) $p(y)=\cos(\alpha y)$.

8. (5%) A string is stretched and secured on the x-axis at $x=0$ and $x=1$ for $t>0$, that is initially held at these points $0.01\sin(3\pi x)$ and then simultaneously released at all points at time $t=0$. The string is released from rest from the initial displacement. (Wave equation constant: a) $u(x,t)=0.01f(x)g(t)$
(A) $g(t)=\cos(\pi at)$,　(B) $f(x)=\sin(3\pi x)$,　(C) $f(x)=\sin(3\pi ax)$,　(D) $g(t)=\cos(3\pi at)$,
(E) $g(t)=\sin(3\pi at)$.

9. (5%) An undamped string/mass system, in which the mass $m=1$ slug and the spring constant $k=10$ lb/ft, is driven by the 2-periodic external force $f(t)=1-t, 0<t<2;\ f(t+2)=f(t)$. Assume that when $f(t)$ is extended to the negative t-axis in a periodic manner, the resulting function is odd. Find a particular solution Xp(t).

$$Xp(t)=\sum_{n=1}^{\infty}\frac{2}{s(10-k)}\sin\,[p(t)]$$

(A) $p(t)=n\pi t$,　(B) $s=n\pi$,　(C) $k=n\pi$,　(D) $k=n^2\pi^2$,　(E) $s=n^2\pi^2$.

10. (5%) The sum of k mutually independent, squared zero-mean, unit-variance Guassian random variables is a chi-square random variable with k degrees of freedom. The equations below are the pdf and moment-generating function of a chi-square random variable Y, respectively, with 6 degrees of freedom. Which of the following are true?

$$f_Y(x)=\frac{x^2\cdot e^{-0.5x}}{16},\quad M_Y(t)=\left(\frac{1}{1-2t}\right)^3.$$

(A) E[Y]=0.
(B) E[Y]=6.
(C) VAR[Y]=6.
(D) VAR[Y]=12.
(E) None of the above.

見背面

國立臺灣大學 106 學年度碩士班招生考試試題

11. (5%) A regular tetrahedron is an object in which all four faces are equilateral triangles. If we need to paint the four faces into four different colors, Red, Green, Blue, and White, how many different color arrangements are available?

(A) 24.
(B) 12.
(C) 6.
(D) 2.
(E) None of the above.

12. (5%) A random sample of 400 interviews on campus showed that the average surfing time on the internet is 71 hours per month. Assuming that the population standard deviation is 10 hours, determine if this indicates that the mean surfing time is greater than 70 hours or not at the 0.05 significance level. Which set of hypotheses is correct?

(A) $H_0: \mu < 70, H_1: \mu \geq 70$.
(B) $H_0: \mu = 70, H_1: \mu > 70$.
(C) $H_0: \mu = 70, H_1: \mu \neq 70$.
(D) $H_0: \mu \geq 70, H_1: \mu < 70$.
(E) None of the above.

13. (5%) According to Problem 12 above, given the critical/rejection region is $z > 1.645$, which of the following decision is correct based on this sample?

(A) Reject H_0.
(B) Accept H_0.
(C) Reject H_0 with probability of 0.95.
(D) Accept H_0 with probability of 0.95.
(E) None of the above.

14. (5%) The probability of a bit error in a transmission line is 10^{-6}. If each bit transmission can be modeled as a Bernoulli trial, estimate the probability (rounded) that a block of 10^6 bits has three or more errors:

(A) 0.08.
(B) 0.16.
(C) 0.33.
(D) 0.67.
(E) None of the above.

接次頁

15. (5%) The random variables X_1, X_2, X_3 have the joint Gaussian pdf:

$$f_{X_1,X_2,X_3}(x_1,x_2,x_3) = \frac{e^{-(0.5x_1^2+2x_2^2-2\sqrt{3}x_2x_3+2x_3^2)}}{\pi^{1.5}\sqrt{2}}.$$

What is the marginal (joint) pdf of X_1 and X_3?

(A) 0.

(B) $\frac{e^{-0.5x_1^2}}{\sqrt{2\pi}} \cdot \frac{e^{-2x_3^2}}{\sqrt{2\pi}}$.

(C) $\frac{e^{-0.5x_1^2}}{\sqrt{2\pi}} \cdot \frac{e^{-0.5x_3^2}}{\sqrt{2\pi}}$.

(D) $\frac{e^{-0.5x_1^2}}{\sqrt{2\pi}} \cdot \frac{e^{-2x_3^2}}{2\pi}$.

(E) None of the above.

16. (5%) According to Problem 15 above, for the 3×3 covariance matrix of (X_1, X_2, X_3), determine the correct element values k_{ij} of the covariance matrix.

(A) $k_{11} = 1$.

(B) $k_{33} = 1$.

(C) $k_{21} = 0$.

(D) $k_{23} = \frac{\sqrt{3}}{2}$.

(E) None of the above.

17. (5%) For a random variable M with pmf given by:

$$P[M = k] = ((0.35)^{k-1} \cdot 0.65), k = 1, 2, \cdots.$$

Which of the following are true?

(A) $\lim_{k\to\infty} P[M \geq k] = 0$.

(B) $P[M > 2] = 0.65$.

(C) $P[M \geq 1001 \mid M > 1000] = 1$.

(D) $P[M \geq 2 \mid M > 1] = 0.65$.

(E) None of the above.

18. (5%) Which of the following statements are true?

(A) If two events are independent, then they must be mutually exclusive.

(B) If two events are mutually exclusive, then they must be independent.

(C) If G and H are independent events, then $P(G|H) = 0$.

(D) If G and H are mutually exclusive events, then $P(G|H) = 0$.

(E) None of the above.

見背面

19. (5%) Suppose that 45% of the students in a class are from Department A, others are from Department B. We also know that 20% of the students from Department A failed the class, and 12% of the students from Department B failed the class. Let F and F^c be the events that the student failed and passed, respectively. Let A and B be the events that the student is from Department A and Department B, respectively. Which of following are true?

(A) $P(A|F) < P(B|F)$.
(B) $P(A|F) < P(A|F^c)$.
(C) $P(B|F) + P(B|F^c) = 1$.
(D) $P(B|F) < P(F|B)$.
(E) None of the above.

試題隨卷繳回