※ 注意：請於試卷內之「非選擇題作答區」依序作答，並應註明作答之大題及小題題號。

1. A teacher wants to partition $3n$ students into n groups of 3 students each.

   (a) (5 points) Suppose that there are two students A and B that must be assigned into the same group. How many different ways can the teacher partition these students?

   (b) (5 points) Suppose that there are four students A, B, C, D such that A and B cannot be assigned into the same group, C and D cannot be assigned into the same group. How many different ways can the teacher partition these students?

   (The ordering of the groups does not matter. For example, partition students $1, 2, 3, 4, 5, 6$ into $(1, 2, 3), (4, 5, 6)$ or $(6, 5, 4), (3, 2, 1)$ are the same.)

2. (10 points) Solve the following recurrence:

$$\begin{aligned} a_0 &= 4, \\ a_1 &= 4, \\ a_n &= 3a_{n-1} - 2a_{n-2} + 1, \text{ for all } n \geq 2. \end{aligned}$$

3. (10 points) Find two positive integers a, b such that 2017a - 100b = 1

4. (40 points) For each of the following statements, determine whether it is true of false. No explanation is needed. You get +5 points for every correct answer and -6 points for every incorrect one. (0 points if you do not answer.)

   (a) $\exists x \exists y P(x, y) \equiv \exists x \exists y P(y, x)$.

   (b) If π is a rational number, then the set of integers is uncountable.

   (c) $\{\leftrightarrow, \neg\}$ is a functionally complete set.

   (d) If S is uncountable and T is countable, then $S \setminus T$ must also be uncountable.

   (e) There exists a injection from the set of all rational numbers to the set of all integers.

   (f) $3^{105} \equiv 1 \pmod{106}$.

   (g) If relations R, S are transitive, then $R \cup S$ is also transitive.

   (h) If relations R, S are transitive, then $R \cap S$ is also transitive.

5. (10 points) Let G be an undirected graph with n vertices and $3n+1$ edges. Does G always have a vertex with degree at least 7? Prove your answer.

6. (20 points) Let G be a simple undirected planar graph with an even number of vertices. Suppose that G and the complementary graph $\bar{G}$ are isomorphic to each other. List all possible values for the number of vertices n. Prove your answer.

試題隨卷繳回