1. Find the solutions of the differential equations:

(1) $y' = \dfrac{y - x}{y - x - 1}$, $y(-5) = -5$ (10%)

(2) $y'' + n^2\pi^2 y = 0, y(0) = 0, y(1) = 0$, n is an integer (10%)

2. Given the vectors $\vec{u} = 3\vec{i} - 2\vec{j} + 4\vec{k}$, $\vec{v} = 3\vec{i} + 2\vec{k}$, $\vec{w} = \vec{j} + 4\vec{k}$
Evaluate $(\vec{u} + \vec{v}) \cdot (\vec{v} + \vec{w}) \times \vec{w}$ (10%)

3. Find the Taylor series of the following functions about the point $Z = a$, and determine the radius of convergence.
(1) $1/(1-Z)$, $a = -1$ (10%)
(2) $\cos Z$, $a = -\pi/2$. (10%)

4. A bar with volume 500m^3 has 50 smokers in it, each smoking two cigarettes per hour. An individual cigarette emits, among other things, about 1.4mg of formaldehyde (HCHO). Formaldehyde converts to carbon dioxide with a reaction rate coefficient K=0.40/hr (first-order reaction). Fresh air enters the bar at the rate of 1000 m^3/hr, and stale air leaves at the same rate. (a) Assuming complete mixing, estimate the steady-state concentration of formaldehyde in the air. (10%); (b) Suppose the air in the bar is clean when it opens at 5 pm. If formaldehyde with reaction rate K=0.40/hr, is emitted from cigarettes smoke at the constant rate of 140 mg/hr starting at 5 pm, what would the concentration be at 6 pm? (10%); (c) At 25℃ and 1 atm of pressure, how does the result compare with the threshold for eye irritation of about 0.05 ppm? (10%)

5. A radioactive substance with decay constant "k" is produced at a constant rate of "r" units of mass per unit time.
(1) Assuming that $Q(0) = Q_0$, find the mass $Q(t)$ of the substance present at time t. (10%)
(2) Find $\lim_{t \to \infty} Q(t)$ (10%)

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