(1) (20 pts) A particle moves in the plane. At time t it is at the point $(x(t), y(t))$, where

$$x(t) = 3\cos t + 3t\sin t, \quad y(t) = 3\sin t - 3t\cos t.$$

At time $t = 0$ the object is at the point $(x(0), y(0)) = (3, 0)$. In this problem assume that t is measured in seconds and that x and y give coordinates in units of centimeters.

(a) (8 pts) Plot the graph of the particles path for $0 \le t \le 10$.

(b) (6 pts) From time $t = 0$, find how long it takes the object to travel a total distance of 24 centimeters, and find the position of the particle at this time.

(c) (6 pts) Find the slope of the line tangent to the graph at the point for time $t = \pi/2$ seconds.

(2) (20 pts) Consider the region inside the curve $r = 2 + \sin 3\theta$ and outside the curve $r = 3 - \sin 3\theta$.

(a) (10 pts) Draw the curves determined by $r = 2 + \sin 3\theta$ and the curve $r = 3 - \sin 3\theta$. Give the polar coordinate pairs where the two curves intersect.

(b) (10 pts) Find the areas of one of these pieces of the region inside the curve $r = 2 + \sin 3\theta$ and outside the curve $r = 3 - \sin 3\theta$.

(3) (10 pts) Let $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ both be convergent series of positive terms. For each of the following decide whether the given series *must always converge*, *must always diverge* or *if it is impossible to tell*. If the series always converges or always diverges, give reasons. If it is impossible to tell, give an explicit example of series $\sum_k a_k$ and $\sum_k b_k$ both convergent, for which the series in question diverge *and* an explicit example of series $\sum_k a_k$ and $\sum_k b_k$ both convergent, for which the series in question converges.

(a) (5 pts) $\sum_{k=1}^{\infty} a_k/b_k$.

(b) (5 pts) $\sum_{k=1}^{\infty} \ln(a_k/b_k)$

(4) (10 pts) Prove that the function $\sqrt{x}$ is uniformly continuous on the interval $[0.01, 100]$ by determining $\delta > 0$ such that $|\sqrt{x} - \sqrt{y}| < \epsilon$ whenever $0.01 \le x < y \le 100$ and $|x - y| < \delta$. In your answer, you should prescribe δ in terms of ϵ.

(5) (20 pts) Let f be a function which is twice differentiable at some point x_0. Define another function by

$$g(x) = \begin{cases} \frac{f(x)-f(x_0)}{x-x_0} & x \ne x_0 \\ f'(x_0) & x = x_0. \end{cases}$$

(a) (10 pts) Is the function $g(x)$ differentiable at x_0? Please give a reason to support your statement.

(b) (10 pts) If $g(x)$ is differentiable, determine $g'(x_0)$.

(6) (20 pts) Determine $\mathrm{c} \in R^3$ of points falling on the unit sphere, $x^2 + y^2 + z^2 = 1$ and the plane, $\mathrm{a}^T\mathrm{x} = b$ where a is a given nonzero unit vector in R^3.

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