國立臺灣大學 103 學年度碩士班招生考試試題

For Questions 1 to 10, select a correct answer for each question and mark the letter (A), (B), (C), or (D) on your answer card. ※ 注意：請用 2B 鉛筆作答於答案卡，並先詳閱答案卡上之「畫記說明」。

1. (5%) Suppose that the relationship between the tax rate t on imported shoes and the total sales S (in millions of dollars) is given by $S(t) = 8 - 15\sqrt[3]{t}$. Find the tax rate t that maximizes revenue for the government.
   (a) 3.6%
   (b) 6.4%
   (c) 9.2%
   (d) None of the above

2. (5%) The capital value of an asset (such as an oil well) that produces a continuous stream of income is the sum of the present values of all future earnings from the asset. Therefore, the capital value of an asset that produces income of $r(t)$ dollars at time t (discounted at a continuous interest rate i) is

   $$\text{Capital Value} = \int_0^T r(t)e^{-it}dt,$$

   where T is the expected life (in years) of the asset. For an oil well that produces income of $r(t) = 10000t^2$ for the following 10 years and the interest rate $i = 5\%$, what is the capital value of this oil well?
   (a) 2,302,028 dollars
   (b) 99,346,934 dollars
   (c) 26,563,254 dollars
   (d) None of the above

3. (5%) What is the solution form of $A(t)$ if
   $$A'(t) = A^2(t) - 1, \text{ and } A(0) = 0.$$
   (a) $A(t) = a + be^{ct}$
   (b) $A(t) = (a + be^{ct})^{-1}$
   (c) $A(t) = (a + be^{ct})^{-1} + d$
   (d) None of the above

4. (5%) Solve the function $A(t)$ in Question (3). What is the value of $A(1)$?
   (a) 1.2976
   (b) -0.2976
   (c) -0.7616
   (d) None of the above

5. (5%) A company's production is given by the Cobb-Douglas function $P = 60L^{2/3}K^{1/3}$, where L and K are the number of units of labor and capital. Each unit of labor costs \$25 and each unit of capital costs \$100. The company wants to produce exactly 1920 units. Find L^* and K^* that meet the production requirements at the lowest cost. What is the value of $L^* + K^*$?
   (a) 64
   (b) 72
   (c) 80
   (d) None of the above

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6. What is the value of $\lim_{n\to\infty}\left(1+\frac{2}{n^2}\right)^n$? (5%)

(a) 2
(b) 1
(c) 0
(d) None of the above

7. What is the value of $\sum_{n=2}^{\infty}\ln\left(1-\frac{1}{n^2}\right)$? (5%)

(a) ∞
(b) $-\ln 1$
(c) $-\ln 2$
(d) $-\infty$

8. What is the value of $\int_0^1\int_0^2 xe^{xy}\,dxdy$? (5%)

(a) e^2-3
(b) $\frac{3}{4}e^4+\frac{1}{4}$
(c) e^2+2
(d) None of the above

9. If a variable x is exponentially distributed with the following probability density function,

$$f(x)=\begin{cases}2e^{-2x} & \text{if } x\geq 0\\ 0 & \text{if } x<0\end{cases}$$

what is the expected value of x? (5%)

(a) e^2
(b) 2
(c) 0.5
(d) None of the above

10. (5%) Find the maximum value of $f(x,y)=\cos x+\cos y$ subject to the constraint condition $y-x=\frac{\pi}{4}$.

(a) $\cos\frac{\pi}{4}+\cos\frac{\pi}{2}$

(b) $\cos\frac{-\pi}{4}+\cos 0$

(c) $\cos\frac{-3\pi}{16}+\cos\frac{\pi}{16}$

(d) None of the above

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For Questions 11 to 15, show your calculations/proof **in detail** on the answer sheet.

11. (10%) Evaluate

$$\lim_{h\to 0^+} h^h$$

12. (10%) Compute

$$\int_1^2 \frac{dx}{x^3(9-x^3)^{1/3}}$$

13. (10%) Calculate $\int_{-\pi}^{\pi} \cos mx \cos nx\, dx$ and $\int_{-\pi}^{\pi} \sin mx \sin nx\, dx$, where m and n are different positive integers.

14. (10%) Maximize the objective function $f(x_1, x_2) = x_1 x_2$ subject to the constraint $x_1 + 4x_2 = 16$.

15. (10%) What is an open set? Show that the interval

$$(0,1) \equiv \{x \in \mathbb{R}: 0 < x < 1\}$$

is an open set.

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