1. (10%) Solve the function $A(t)$ if

$$A'(t) = \alpha A^2(t) + \beta A(t) - 1, \text{ and } A(0) = 0,$$

where α and β are real numbers.

2. (10%) Given

$$y_m = \int \frac{dx}{(x^2+4)^m},$$

express y_m as $A + By_{m-1}$, where A and B are functions of x and m.

3. (10%) Find the Taylor series about $x = 0$ for the following integral:

$$\int x^2 e^{-x^2} dx.$$

4. (20%) The Black-Scholes formula for a call option with six input parameters (S, X, r, q, σ, T) is as follows.

$$c(S, X, r, q, \sigma, T) = Se^{-qT}N(d_1) - Xe^{-rT}N(d_2),$$

where

$$d_1 = \frac{\ln(S/X) + (r - q + \sigma^2/2)T}{\sigma\sqrt{T}} \text{ and } d_2 = \frac{\ln(S/X) + (r - q - \sigma^2/2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T},$$

and $N(\cdot)$ is the cumulative distribution function of the standard normal distribution defined as

$$N(d) = \int_{-\infty}^{d} n(x)dx = \int_{-\infty}^{d} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx,$$

where $n(\cdot)$ is the probability density function of the standard normal distribution.

(a) (5%) Derive and express $\frac{\partial c}{\partial S}$ as the form of $e^A N(B)$. What are A and B?

(b) (5%) Derive and express $\frac{\partial^2 c}{\partial S^2}$ as the form of $\frac{n(C)e^D}{E}$. What are C, D, and E?

(c) (5%) Derive and express $\frac{\partial c}{\partial \sigma}$ as the form of $Fe^G n(H)$. What are F, G, and H?

(d) (5%) Derive and express $\frac{\partial c}{\partial r}$ as the form of $Ie^J N(K)$. What are I, J, and K?

(Please write down the detailed calculation process.)

見背面

國立臺灣大學 102 學年度碩士班招生考試試題

5. (10%) Represent $(1-x)^{-2}$ in a Maclaurin series for $-1 < x < 1$.

6. (10%) Find the equation of the line tangent to the curve $x = 2t^3 - 15t^2 + 24t + 7$, $y = t^2 + t + 1$ at $t = 2$.

7. (10%) Find the maximum and minimum values of $f(x,y) = xy^2$ subject to the condition $x^2 + y^2 = 1$.

8. (10%) Evaluate $\int (\tan^5 x)(\sec^4 x)dx$.

9. (10%) Determine the interval of convergence of $\sum_{n=1}^{+\infty} \frac{x^n}{2+n^2}$.