1. [10 分] What are the meanings of (a) the power spectral density, (b) the Gibbs phenomena, and (c) the aliasing effect?
2. [8 分] Determine the Fourier series of $x(t)$

$$x(t)=\begin{cases}1-t & \text{for } 0\le t<1\\ 1+t & \text{for } -1\le t<0\end{cases},$$
$$x(t)=x(t+2).$$

3. [8 分] Calculate $\sum_{n=-\infty}^{\infty}\frac{\sin^2(n)}{\pi^2 n^2}$
4. [8 分] Suppose that $y[n] = x[3n]$. If the discrete-time Fourier transforms of $x[n]$ and $y[n]$ are $X(e^{j\omega})$ and $Y(e^{j\omega})$, respectively, what are the relation between $X(e^{j\omega})$ and $Y(e^{j\omega})$?
5. [8 分] Calculate the inverse Z transform of $X(z)=\log\left(1+2z^{-1}\right)$
6. [8 分] Suppose that $x(t)$ is a wide-sense stationary random process and $y(t)$ is the convolution of $x(t)$ and $h(t)$ ($h(t)$ is deterministic). Prove that $y(t)$ is also a wide-sense stationary random process.
7. Let X be a random variable with probabilities

$$P(X=k)=\begin{cases}(1-p)p^k, & k=0,1,2,\ldots,l\\ \dfrac{(1-p)p^k}{1-p^m}, & k=l+1,l+2,\ldots,l+m\end{cases}$$

where p is a real number, $0<p<1$, and m is a positive integer.

(a). [4 分] Verify that the probability distribution given for X is valid.

(b). [8 分] Let m satisfy $p^m+p^{m+1}\le 1<p^{m-1}+p^m$, construct a binary Huffman code for a source whose outputs are i.i.d. and distributed like X.

(c). [6 分] If $l\to\infty$ and $p\le 0.5$, what will the binary Huffman code be? What is its average codeword length?

8. Let X be a memoryless Laplacian source with marginal pdf

$$f_X(x)=\frac{1}{2\lambda}e^{-|x|/\lambda},\quad \lambda>0.$$

If we consider 1-bit scalar quantization with an absolute distortion measure $d(x,\hat{x})=|x-\hat{x}|$, where $\hat{x}$ is the quantized value of x,

(a). [8 分] What are the optimum input and output quantization levels?

(b). [4 分] What is the average distortion?

(c). [4 分] It is known that the rate-distortion function for a memoryless Laplacian source with an absolute distortion measure is

$$R(D)=\begin{cases}\log(\lambda/D), & 0\le D\le\lambda\\ 0, & D>\lambda\end{cases}$$

With the same average distortion, up to how much average rate can you save from the quantization in (a) if vector quantization is allowed?

9. A certain source can be modeled as a stationary zero-mean Gaussian process $X(t)$ with power-spectral density

$$S_x(f)=\begin{cases}2, & |f|<10\\ 0, & \text{otherwise}\end{cases}$$

The distortion in reproducing $X(t)$ by $\hat{X}(t)$ is $D=E\left|X(t)-\hat{X}(t)\right|^2$. This source is to be transmitted over an additive Gaussian noise channel, in which the noise power-spectral density is given by

$$S_n(f)=\begin{cases}1, & |f|<4\\ 0, & \text{otherwise}\end{cases}$$

(a). [4 分] Find the rate-distortion function for the source.

(b). [4 分] If we want to reproduce $X(t)$ with a distortion equal to 10, what transmission rate is required?

(c). [8 分] What is the required power such that the source can be transmitted via the channel with a distortion not exceeding 10?

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