1. (20pts) Consider the equation $\sum_{k=1}^{n}\frac{1}{x_k^2}=1$. For which n such that the equation has positive integer solution of x_k? Note: it is known that $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{9}+\frac{1}{9}+\frac{1}{36}=1$ and $\frac{1}{4}+\frac{1}{4}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{36}+\frac{1}{36}=1$

2. (15pts) Solve a_n, where $a_1=1$, $a_2=3$, $a_n=\frac{2}{5}a_{n-1}+\frac{3}{5}a_{n-2}$ for $n\geq 3$. Show details of your derivations.

3. (15pts) There are n ($n\geq 2$) people in a party. Suppose that two persons either know each other or not (A knows B if and only if B knows A). Prove or disprove that there always exist two persons who know exactly the same number of people in the party.

4. (20pts) Let G be a simple connected planar graph containing no triangle. Prove or disprove that G contains at least one vertex of degree 3 or less by Euler's polyhedron formula.

5. (15pts) Show that $\sum_{k=0}^{k=n}C(n,k)^2=C(2n,n)$, where $C(n,k)$ is the coefficient of the x^k term in the expansion of $(1+x)^n$.

6. (15pts) Prove that a relation R is transitive if and only if $R^n\subseteq R$ for all positive integer n.

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