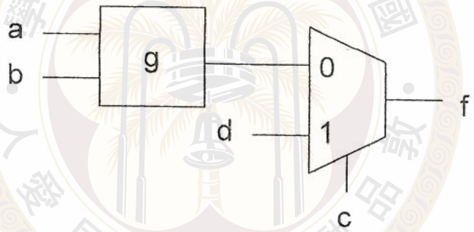
1. (10%) Let $(N)_R$ denote the number N under radix R, and omit specifying the radix when $R = 10$. What should be the number in radix 10 corresponding to $(11)_{(11)_{(11)_{(11)_2}}}$? Please re-express this number in radix 3.

2. (10%) Consider a Boolean function f such that $f = x_2'x_3 + x_2x_4$ when $x_1 = 0$, $f = x_3x_4'x_5$ when $x_1 = 1$ and $x_2 = 0$, and $f = x_3x_5 + x_4'$ when $x_1 = 1$ and $x_2 = 1$. Express f in minimum sum of products.

3. (20%) Suppose a Boolean function f is implemented by the following circuit with a multiplexor and some unknown function g.



In addition, f is partially known as shown in the following Karnaugh map.

cd \ ab	00	01	11	10
00	?	?	1	?
01	1	?	?	0
11	?	?	?	?
10	?	?	?	?

What are the possible functions of f in terms of **minimized** sum of products? What are the corresponding g functions?

4. (15%) There are five fruits F_1, F_2, F_3, F_4, and F_5, whose nutrients are summarized in the following table, where a "√" indicates the availability of some vitamin in some fruit. (For example, fruit F_1 contains vitamin A but not vitamin B.)

	F_1	F_2	F_3	F_4	F_5
Vitamin A	√		√		
Vitamin B			√		√
Vitamin C	√	√			√
Vitamin D			√	√	
Vitamin E		√		√	

(a) (5%) Please write down a product-of-sums formula (in terms of Boolean variables F_1, F_2, F_3, F_4, and F_5) such that it is true for some {0, 1}-assignment to the Boolean variables if and only if all vitamins are acquired by the corresponding collection of fruits with $F_i = 1$.

(b) (5%) Rewrite the formula of (a) to a minimum sum-of-products expression (in an alphabetical order).

(c) (5%) Suppose we want to buy as few fruits as possible while acquiring all vitamins. What fruits should we buy? How does this problem relate to (b)?

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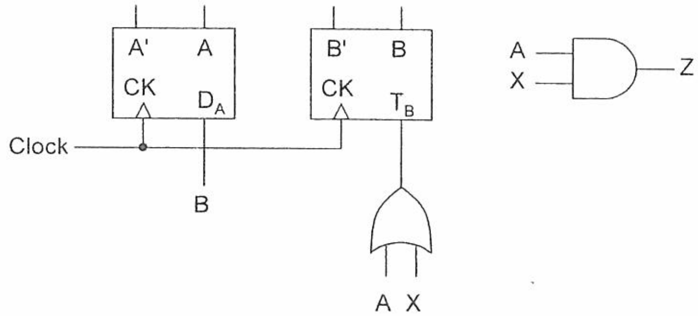
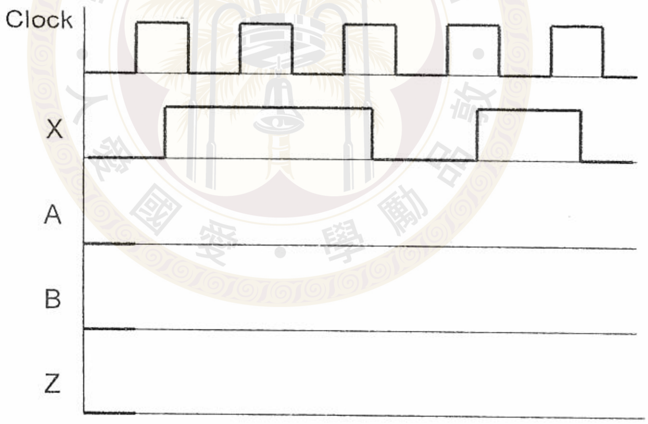
5. (15%) Consider the following circuit, which in part consists of input X, output Z, a D flip flop, and a T flip flop. Assume A = B = 0 initially.



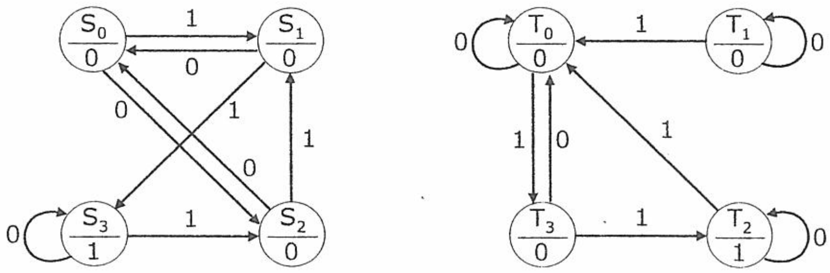
(a) (5%) Plot its state graph. (Let AB = 00 be state S_0, 01 be S_1, 10 be S_2, and 11 be S_3.)

(b) (5%) Complete the following timing chart by assuming negligible propagation delays in flip flops and negligible gate delays. (That is, assume these delays are close to zero compared to those of X).



(c) (5%) Suppose both of the flip flops have propagation delay 2ns and setup time 2ns, and all the gates have propagation delay 3ns. What should be the minimum clock period of the circuit? (Assume X always settles to its correct value within 2ns.)

6. (20%) Let C_1, C_2, and C_3 be three sequential circuits with states $\{S_0, S_1, S_2, S_3\}$, $\{T_0, T_1, T_2, T_3\}$, and $\{S_0, S_1, S_2, S_3, T_0, T_1, T_2, T_3\}$, respectively, whose transitions are shown in the figure below.



(a) (10%) How many non-equivalent states are there in C_1, C_2, and C_3? Which states are equivalent in C_1, C_2, and C_3?

(b) (5%) If C_1 and C_2 start from states S_0 and T_0, respectively, are these two circuits equivalent (in terms of input-output behavior)? If yes, briefly explain why. Otherwise, identify an input sequence that distinguishes C_1 and C_2.

(c) (5%) Redo (b) for C_1 and C_2 starting from states S_0 and T_1, respectively.

7. (10%) For a Moore finite-state machine with one (binary) input and one (binary) output, is there an upper bound for the number of non-equivalent states of the FSM? If yes, what is the upper bound? If no, please construct a state graph with *n* **non-equivalent** states, where *n* is extensible to an arbitrarily large integer.