國立臺灣大學99學年度碩士班招生考試試題

※ 注意：請於試卷內之「選擇題作答區」依序作答。

選擇題 (單選； 每題答對得 2 分; 答錯或未答得 0 分)

1. Suppose the variable x represents students, $F(x)$ means "x is a freshman", and $M(x)$ means "x is a math major". What does the formula $\neg\forall x(\neg F(x) \vee \neg M(x))$ mean:
   (A) Some freshmen are math majors.
   (B) Every math major is a freshman.
   (C) No math major is a freshman.
   (D) Some freshmen are not math majors.
   (E) None of the above.

2. Which of the following statements is not correct?
   (A) Propositional formula $(A \wedge B)\vee(\neg A)\vee(\neg B)$ is a tautology.
   (B) Propositional logic does not have a sound and complete deduction system.
   (C) First-order logic is more powerful than propositional logic.
   (D) "2+2=0" is a proposition.
   (E) Propositional formula $p \rightarrow (q \rightarrow r)$ is satisfiable.

3. From the following four,
   (1) $(p \rightarrow q) \rightarrow r$
   (2) $p \rightarrow (q \rightarrow r)$
   (3) $q \rightarrow (p \rightarrow r)$
   (4) $(q \rightarrow p) \rightarrow r$
   which are the two propositions that are logically equivalent?
   (A) 1 and 2 (B) 1 and 3 (C) 2 and 3 (D) 2 and 4 (E) None of the above

4. Using c for "it is cold", r for "it is rainy", and w for "it is windy", which of the following propositional formula means "It is rainy only if it is windy and cold" in symbols?
   (A) $r \rightarrow (w \wedge c)$
   (B) $\neg r \rightarrow (w \vee \neg c)$
   (C) $(w \wedge c) \rightarrow r$
   (D) $r \wedge (w \wedge c)$
   (E) None of the above

5. Let predicate $P(m,n)$ mean "$m \leq n$", where the universe of discourse for m and n is the set of nonnegative integers. Among the following four first-order formulas, how many of them are true?
   $\exists nP(n,0)$; $\forall nP(0,n)$; $\exists n\forall mP(m,n)$; $\forall m\exists nP(m,n)$,
   (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

6. Consider the following five sets: N (the set of natural numbers), Z (the set of integers), Q (the set of rational numbers), R (the set of real numbers), and 2^N (the power set of N), how many of the above five sets are countably infinite?
   (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

7. Define predicate $P(x, y)$ to be "$x + 2y = xy$", where x and y are integers. How many of the following four first-order formulas $\forall x\exists yP(x,y)$, $\exists x\forall yP(x,y)$, $\forall y\exists xP(x,y)$, $\exists y\forall xP(x,y)$ are "true"?
   (A) 0 (B) 1 (C) 2 (D) 3 (E) 4

8. Let S={∅, {∅, {∅}}}. What is 2^S (i.e., the power set of S)?
   (A) {(∅, ∅), (∅, {∅}), ({∅}, ∅), ({∅}, {∅})}
   (B) {∅, {∅}, {{∅}}, {∅, {∅}}}
   (C) {∅, {∅, {∅}}}
   (D) {∅, {∅}, {{∅}}}
   (E) None of the above

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9. Let S={∅, {∅}}. What is the value of |S×{∅}| ?
(A) 0 (B) 1 (C) 2 (D) 3 (E) None of the above

10. Consider a binary relation S={(*a*, *c*), (*b*, *d*), (*d*, *a*)}. What is S^3?
(A) {(*a*, *a*), (*a*, *c*), (*b*, *c*), (*c*, *c*), (*d*, *b*), (*d*, *d*)}
(B) {(*b*, *c*)}
(C) {(*a*, *c*), (*b*, *a*), (*d*, *b*), (*d*, *d*)}
(D) {(*a*, *a*), (*a*, *d*), (*d*, *c*)}
(E) None of the above

11. Define a binary relation *R(x,y)* on the set of real numbers as: "$x = y + 1$ or $x = y - 1$". Consider the following four properties: *reflexive, symmetric, anti-symmetric*, and *transitive*. *R(x,y)* satisfies how many of the above four properties?
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

12. Suppose $|A| = n$. What is the number of symmetric binary relations on *A*?
(A) $2^{n(n-1)/2}$ (B) $2^{n(n+1)/2}$ (C) 2^n (D) $2^{n(n-1)}$ (E) None of the above

13. Suppose $|A| = n$. What is the number of reflexive, symmetric binary relations on *A*.
(A) $2^{n(n-1)/2}$ (B) $2^{n(n+1)/2}$ (C) 2^n (D) $2^{n(n-1)}$ (E) None of the above

14. If $R = \{(1,2),(1,4),(2,3),(3,1),(4,2)\}$, what is the size (i.e., the number of elements) of the symmetric closure of *R*?
(A) 8 (B) 9 (C) 10 (D) 11 (E) None of the above

15. The symmetric difference of two sets *A* and *B* (denoted by $A \oplus B$) is the set of elements which are in *A* or *B*, but not in both. How many of the following four statements are correct?
-- $\{1, 3, 5\} \oplus \{1, 2, 3\} = \{2,5\}$
-- $A \oplus B = (A \cup B) - (A \cap B)$
-- $A \oplus (B \oplus C) = (A \oplus B) \oplus C$
-- $(A \oplus B) \oplus B = A$
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

16. Consider $f: N \to Z$ where

$$f(n) = \begin{cases} \dfrac{-n}{2}, & n \text{ even} \\ \dfrac{n-1}{2}, & n \text{ odd} \end{cases}$$

Which of the following is correct?
(A) *f*(n) is onto but not 1-1
(B) *f*(n) is onto and 1-1
(C) *f*(n) is 1-1 but not onto
(D) *f*(n) is not onto and not 1-1
(E) *f*(n) is not a function.

17. How many permutations of the seven letters *A*,*B*,*C*,*D*,*E*,*F*,*G* have *A* immediately to the left of *E*?
(A) $2 \cdot 5!$ (B) $6!$ (C) $5 \cdot 6!$ (D) $7! - 2 \cdot 6!$ (E) None of the above

18. How many one-to-one functions are there from a set with three elements to a set with eight elements?
(A) 512 (B) 336 (C) 128 (D) 270 (E) None of the above

19. What is the largest coefficient in the expansion of $(x + 1)^6$?
(A) 6 (B) 15 (C) 20 (D) 30 (E) None of the above

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20. Suppose $g: A \to B$ and $f: B \to C$ where $A = \{1,2,3,4\}$, $B = \{a,b,c\}$, $C = \{2,8,10\}$, and g and f are defined by $g = \{(1,b),(2,a),(3,b),(4,a)\}$ and $f = \{(a,8),(b,10),(c,2)\}$. What is $f \circ g$?
(A) $\{(2,c),(8,a),(10,b)\}$
(B) $\{(2,2),(8,8),(10,10)\}$
(C) $\{(1,10),(2,8),(3,10),(4,8)\}$
(D) $\{(1,1),(2,1),(3,2),(4,1)\}$
(E) None of the above

21. What is the value of 50! mod 49! ?
(A) 0　(B) 1　(C) 25　(D) 48　(E) None of the above

22. What is the number of times that "hello" is printed after the following program is executed?

```
i := 1, j := 1
while i ≤ n
 begin
  while j ≤ i
    begin
    print ``hello";
    j := j + 1
    end
  i := i + 1
 end.
```

(A) $\Theta(1)$　(B) $\Theta(n)$　(C) $\Theta(n^2)$　(D) $\Theta(n\log_2 n)$　(E) None of the above

23. Which of the following functions has the largest growth rate asymptotically?
(A) $0.1n^{1.5} + 3n$　(B) $\frac{n^2}{\log n}$　(C) $\sqrt{n}$　(D) $(0.001)^n$　(E) $(\log n)^2$

24. Consider a binary relation R on $\{1,2,3,4\}$ whose matrix representation is $\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$.
For the following four properties: *reflexive, symmetric, anti-symmetric*, and *transitive*, R satisfies how many of them?
(A) 0　(B) 1　(C) 2　(D) 3　(E) 4

25. Given a partial order relation R with its Hasse diagram shown in Figure 1. What is $lub(\{g, j, m\})$?
(A) $\{l, m\}$
(B) $\{m\}$
(C) $\{l\}$
(D) $\{g, j, m\}$
(E) None of the above

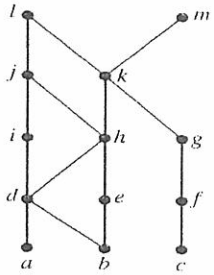

Figure 1

26. Consider Figure 1 again. What is the set of least elements of R?
(A) $\{a\}$
(B) $\{a, b, c\}$
(C) $\{c\}$
(D) $\varnothing$
(E) None of the above

27. Consider relation R in Figure 1 again. Which of the following is false?
(A) aRa
(B) eRm
(C) R does not have a greatest element
(D) R is a lattice
(E) None of the above

28. The chromatic polynomial $\chi_G(k)$ of a graph G is the number of proper colorings of G with k colors. Consider graph G in Figure 2. What is $\chi_G(k)$?
(A) $k(k-1)(k-2)(k-3)$
(B) k^4
(C) $k^2(k-1)^2$
(D) $k(k-1)^3$
(E) None of the above

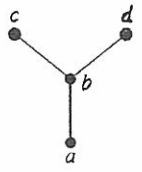

Figure 2

29. What is the number of positive integers not exceeding 100 (i.e., $\leqq 100$) that are not divisible by 5 or by 7?
(A) 65　(B) 66　(C) 67　(D) 68　(E) None of the above

30. What is the transitive closure of $R=\{(1,1), (2,2), (3,3), (2,3), (3,1)\}$?
(A) $\{(1,1), (2,2), (3,3), (2,3), (3,1)\}$
(B) $\{(1,1), (2,1), (2,2), (3,3), (2,3), (3,1)\}$
(C) $\{(1,1), (2,2), (3,3)\}$
(D) $\varnothing$
(E) None of the above

31. Which of the following statements is correct?
(A) If $A \cup C = B \cup C$, then $A = B$
(B) If $A \cap C = B \cap C$, then $A = B$.
(C) $\{\varnothing, \{a\}, \{\varnothing, a\}\}$ is the power set of some set.
(D) $\varnothing \subseteq \{\varnothing\}$
(E) $A \cap (B \cup C) = (A \cup B) \cap (A \cup C)$.

32. Which of the following statements is correct?
(A) The union of an infinite number of countably infinite sets is always countably infinite.
(B) $A \times B$ is always countably ininfite, if both A and B are countably infinite.
(C) If $f: A \rightarrow B$ and B is countable infinite, then A is always countably infinite.
(D) If A is countably infinite, then 2^A is always countably infinite
(E) None of the above is correct.

33. For complete bipartite graph $K_{m,n}$ to be planar, what is the maximum value of $m+n$?
(A) 5　(B) 6　(C) 7　(D) 8　(E) None of the above

34. If $f: X \rightarrow Y$ is a one-to-one and onto function, and Y is a proper subset of X, which of the following statements is correct?
(A) X must be countably infinite.
(B) Y must be countably infinite.
(C) X must be infinite.
(D) The cardinality of X is larger than Y.
(E) None of the above

35. What is the number of distinct binary trees with 4 nodes?
(A) 10　(B) 12　(C) 14　(D) 16　(E) None of the above

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36. Let (S, $\lesssim$) be a POSET. (S, $\lesssim$) is well-ordered if $\lesssim$ is a total ordering such that every nonempty subset of S has a least element according to $\lesssim$. Which of the following is well-ordered (here $\leqq$ denotes the "less than or equal to" relation in arithmetic)?
(A) (Z, $\leqq$), where Z is the set of integers
(B) (Q, $\leqq$), where Q is the set of rational numbers
(C) (N×N, $\lesssim$), where $(a, b) \lesssim (c, d)$ if and only if $a \leqq c$ and $b \leqq d$, where N is the set of natural numbers
(D) ([0,1], $\leqq$), where [0,1] denotes the set $\{x \mid 0 \leqq x \leqq 1, x \text{ is a real number}\}$
(E) None of the above

37. If a planar graph has 12 vertices, each of degree 3, how many edges and faces does the graph have?
(A) number of edges = 18; number of faces = 7;
(B) number of edges = 18; number of faces = 8;
(C) number of edges = 36; number of faces = 25;
(D) number of edges = 36; number of faces = 26;
(E) None of the above

38. Consider the following recurrence relation: $f(n+2)-3f(n+1)+2f(n)=2^n$ subject to $f(0)=1, f(1)=4$. Suppose we know that the solution of the above recurrence relation is of the form $f(n)= an2^n+b2^n+c$, where a, b, and c are three constants. What is the value of $a+b+c$?
(A) 1 (B) 1.5 (C) 2.5 (D) 3 (E) None of the above

39. Among the four graphs displayed in Figure 3, how many of them are planar?
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

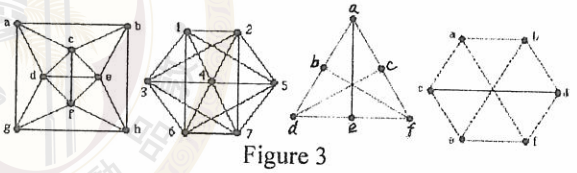

Figure 3

40. Which of the following is a possible sequence of vertex degrees of an undirected simple graph?
(A) 2, 2, 2, 3, 4, 4
(B) 0, 1, 2, 3, 4, 5, 6, 7
(C) 1, 1, 2, 4
(D) 0, 1, 2, 2, 3
(E) None of the above

41. Consider the graph G shown in Figure 4. Which of the following is true:
(A) G has an Euler circuit but no Hamilton circuit.
(B) G has an Euler path but no Euler circuit.
(C) G has an Euler circuit and a Hamilton circuit.
(D) G has no Euler circuit and no Hamilton circuit.
(E) None of the above

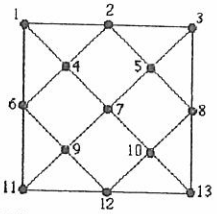

Figure 4

42. How many solutions does the equation $x_1+x_2+x_3=13$ have, where x_1, x_2, x_3 are nonnegative integers less than 6?
(A) 5 (B) 6 (C) 7 (D) 8 (E) None of the above

43. Which one is the smallest partial order relation on {1,2,3} that contains (3,2), (1,3)?
(A) { (3,2), (1,3)}
(B) {(1,1),(2,2),(3,3),(3,2),(2,3),(1,3),(3,1),(1,2), (2,1)}
(C) {(1,1),(2,2),(3,3),(3,2),(1,3),(1,2)}
(D) {(1,1), (3,2), (1,3),(2,3),(3,1)}
(E) None of the above

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44. Consider the following four statements:
If $a \equiv b \pmod m$, and $a \equiv c \pmod m$, then $a \equiv b + c \pmod m$.
If $a \equiv b \pmod m$ and $c \equiv d \pmod m$, then $ac \equiv b + d \pmod m$.
If $a \equiv b \pmod m$, then $2a \equiv 2b \pmod m$.
If $a \equiv b \pmod m$, then $2a \equiv 2b \pmod{2m}$
how many of them are correct?
(A) 0 (B) 1 (C) 2 (D) 3 (E) 4

45. Let Q_n denote the n-dimensional hypercube. (Q_3 is shown in Figure 5.)
What is the number of edges of Q_n?
(A) $2^{n+1}-4$ (B) $n2^{n-1}$ (C) $(n+1)n$ (D) $4n$ (E) None of the above

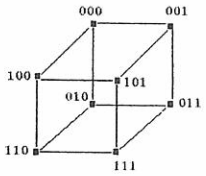

Figure 5

46. Which of the following recurrence relations characterizes the so-called Fibonacci numbers:
(A) $a_n = 2a_{n-1} - a_{n-2}$, $a_1 = 1, a_2 = 1$.
(B) $a_{n-1} = a_n + a_{n-2}$, $a_1 = 1, a_2 = 1$.
(C) $a_n = a_{n-1} + a_{n-2}$, $a_1 = 1, a_2 = 1$.
(D) $a_n = na_{n-1}$, $a_1 = 1$.
(E) None of the above

47. Given two functions $g: A \to B$ and $f: B \to C$, which of the following is not correct:
(A) If both f and g are onto, so is $f \circ g$.
(B) If both f and g are one-to-one, so is $f \circ g$.
(C) If both f and $f \circ g$ are one-to-one, g need not be on-to-one.
(D) If both g and $f \circ g$ are onto, f must be onto.
(E) $f \circ g$ is a function from A to C.

48. Given a finite set S, consider $(2^S, \subseteq)$. Which of the following is not correct:
(A) $\varnothing$ is the least element.
(B) S is the greatest element
(C) $\subseteq$ is a total ordering
(D) $(2^S, \subseteq)$ is a lattice.
(E) $A \cup B$ is the least upper bound of $\{A, B\}$, where $A \subseteq S$ and $B \subseteq S$.

49. Which of the following is true?
(A) $A - (B - C) = (A - B) - C$.
(B) $(A - C) - (B - C) = A - B$.
(C) $A \cup (B \cap C) = (A \cap B) \cup (A \cap C)$.
(D) $\overline{A \cup \overline{B}} \cup \overline{A} = \overline{A}$
(E) If $A - C = B - C$, then $A = B$.

50. What is the value of $\sum_{j=1}^{3} \sum_{i=1}^{j} ij$.?
(A) 22 (B) 23 (C) 24 (D) 25 (E) None of the above

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