第壹大題選擇題考生請作答於『選擇題作答區』。」

第壹大題（50%） 每題五分五個選項，其中至少有一個是正確的答案。各題獨立計分，每答對一個選項，得一分；每答錯一個選項時，倒扣一分。每題倒扣至零分為止。不作答者，得零分。

1. Consider the following algorithm:

Input: a sequence $\{b_0, b_1, b_2, ..., b_{n-1}\}$ of boolean elements, all initially **false**.
1. For each $i = 2$ to $n - 1$, set b_i = **true**.
2. Let $p = 2$.
3. Repeat steps 4-5 while $p^2 < n$.
4. For each $j = 2$ to $\lfloor (n-1)/p \rfloor$, set $b_{j \times p}$ = **false**.
5. Increment p until b_p is **true**.

Which of the following statement(s) is (are) true?

(A) The first for loop (line 1) iterates $\Theta(n)$ times.

(B) The second outside loop (line 3) also iterates $\Theta(n)$ times.

(C) The time complexity of this algorithm is $\Theta(n^3)$.

(D) The time complexity of this algorithm is $\Theta(n^2)$.

(E) At the end, b_i is true if and only if i is prime.

2. Consider the following sequence of operations for an empty binary search tree: insert(7), insert(4), insert(3), insert(2), insert(9), insert(1), and insert(6). Which of the following statement(s) is (are) true?

(A) The height of the tree is 4 (The height of a tree with only the root node is 1).

(B) The result of postorder traversal is 1236497.

(C) The result of inorder traversal is 1234697.

(D) After the operation of delete(7), the result of preorder traversal could be 123649.

(E) After the operation of delete(7), the result of preorder traversal could be 123496.

3. Which of the following statement(s) is (are) true?

(A) The access time of a hash table with closed addressing does not related to the load factor.

(B) The access time of a hash table with open addressing does not related to the load factor.

(C) Linear probing suffers from primary clustering.

(D) Quadratic probing does not suffer from secondary clustering.

(E) Rehashing suffers from primary clustering.

4. Which of the following statement(s) about "trees and undirected graphs" is (are) true?

(A) Any undirected graph without cycle is a tree.

(B) Any undirected graph with n nodes and $(n-1)$ edges, where $n \geq 1$, is a tree.

(C) Any connected undirected graph without cycle is a tree.

(D) Any undirected graph where every node connects to at least one other node is a tree.

(E) None of the above.

5. Which of the following statement(s) about the "binary min heap" is (are) true?

(A) A binary min heap is also a binary search tree.

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(B) Every subtree of a binary min heap is also a binary min heap.
(C) Initializing a binary min heap of n elements takes $\Theta(n)$ time over average.
(D) Extracting the minimum from a binary min heap takes $\Theta(1)$ time on average.
(E) Inserting an element into a binary min heap with n elements takes $\Theta(n)$ time in the worst case.

6. Starting from empty, we insert 1, 2, 3, 4, 5, 6, and 7 (in that exact order) to an AVL tree. Which of the following statement(s) is (are) true?
(A) The root is 3.
(B) The height of the tree is 4. (The height of a tree with only the root node is 1.)
(C) The result of postorder traversal is 1 3 2 5 7 6 4.
(D) The result of inorder traversal is 1 2 3 4 5 6 7.
(E) The result after all insertion is a perfect tree.

7. Which of the following statement(s) about "red-black tree" is (are) true?
(A) In a red-black tree, every red node must have two black children.
(B) In a red-black tree, every black node must have two red children.
(C) In a red-black tree, every path from the root to any external nodes contains same number of black nodes.
(D) A red-black tree with n nodes (including external) contains exactly $(n+1)/2$ external nodes.
(E) In a red-black tree, the number of rotations for one insertion is $\Theta(1)$ in the worst case.

8. Initially, each set contains one unique element. Which of the following statement(s) about the "union-and-find algorithm" on the disjoint-set forest is (are) true?
(A) In the best case, finding an element in a set of size n takes $\Theta(\log n)$ time.
(B) In the worst case, finding an element in a set of size n takes $\Theta(n)$ time.
(C) If we always make the taller tree as a subtree of the shorter during union, the height of the tree with n elements is at least $\Theta(n)$.
(D) If we always make the shorter tree as a subtree of the taller during union, the height of the tree with n elements is at most $\Theta(\log n)$.
(E) u unions and f finds can be done in $\Theta(u+f)$ time.

9. Suppose sparse matrices are on average of size n by n with m non-zero entries ($n<m<n^2$), and we store a sparse matrix by n lists: one list per row. Which of the following statement(s) is (are) true?
(A) One such sparse matrix takes $\Theta(n+m)$ space.
(B) Accessing an entry takes $\Theta(nm)$ time on average.
(C) Accessing an entry takes $\Theta(nm)$ time in the worst case.
(D) Accessing all entries of a row takes $\Theta(m)$ time on average.
(E) Accessing all entries of a column takes $\Theta(m)$ time on average.

10. Consider a hash table of 13 buckets, labeled from 0 to 12, and every of which holds only one key. The hash function is $f(key) = key$ % 13, where "%" denotes the modulo operator (for example, (20 % 13) is 7). We use the **linear probing scheme** ($f(key) = (key+i)$ % 13, where i is incremented from 0 for collisions) and insert 5, 9, 24, 44, 50, 10, 6, 41, 11, 47, and 31 (in that exact order) into the hash table.

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Which of the following statement(s) is (are) true?

(A) 31 is stored at bucket 1.

(B) 50 is stored at bucket 11.

(C) 6 is stored at bucket 7.

(D) Finding 31 takes 3 look-ups.

(E) If we delete 44, 11 will be stored at bucket 12.

第貳大題 (50%) ※ 注意：以下各題請於試卷內之「非選擇題作答區」作答，並應註明作答之題號。

11. (10%) Sort the following functions in ascending order (slowest-growth first); put equal signs when several terms are equal. For example, A, B=C, D. Note that there are 10 terms. You don't need to explain your answers.

$$\Theta\left(\frac{n}{\log n}\right), \Theta(n), \Theta\left(\sqrt{n}\right), \Theta\left(n^2\right), \Theta\left((\log n)^2\right), \Theta\left(\log\left(n^2\right)\right), \Theta(n+\log n), \Theta\left(\frac{\sqrt{n^2+1}}{\sqrt{n}}\right), \Theta\left((\log n)^n\right), \Theta\left(n^{\log n}\right)$$

12. (10%) Stacks & Queues

(a) (5%) Starting from an empty stack, show the stack after every following operations: push(2.0), push(5.0), push(4.0), push(pop()/pop()),push(pop()/pop()). You need to note the top and bottom of the stack.

(b) (5%) Repeat (a) but starting with an empty queue: enqueue(2.0), enqueue(5.0), enqueue(4.0), enqueue(dequeue()/dequeue()), enqueue(dequeue()/dequeue()). You need to note the front and rear of the queue.

13. (20%) Binary Search Trees (the height of a tree with only the root node is 1)

(a) (5%) The post-order traversal sequence of a binary search tree is 2 5 7 15 37 50 40 35 20 10. Please draw the binary search tree.

(b) (5%) What is the main purpose to balance a binary search tree?

(c) (5%) Draw an AVL-tree of height 5 with the fewest possible nodes.

(d) (5%) Draw a red-black tree of height 5 with the fewest possible nodes (You need to note colors of nodes).

14. (10%) Below is an adjacency matrix of an undirected graph with 9 vertices labeled from 1 to 9. The entry '1' represents the existence of an edge between two nodes.

	1	2	3	4	5	6	7	8	9
1	0	1	0	1	0	0	0	0	0
2	1	0	0	0	1	0	0	0	0
3	0	0	0	0	1	0	0	1	1
4	1	0	0	0	1	1	1	0	0
5	0	1	1	1	0	0	1	1	0
6	0	0	0	1	0	0	1	0	0
7	0	0	0	1	1	1	0	0	0
8	0	0	1	0	1	0	0	0	1
9	0	0	1	0	0	0	0	1	0

(a) (5%) Starting from Node 1, show the visiting sequence of breadth-first search on the above graph. **Choose the node with a smaller number for multiple choices.**

(b) (5%) Starting from Node 1, show the visiting sequence of depth-first search on the above graph. **Choose the node with a smaller number for multiple choices.**