

1. (15%) Find, if possible, an equation of a plane that contains the following given points: (0, 1, 0); (0, 1, 1) and (1, 3, -1).

2. (20%) For the given matrix

$$A = \begin{pmatrix} 0 & -1 & 0 \\ -1 & -1 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

- (a) Find the eigenvalues.
 (b) Find the corresponding eigenvectors.
 (c) Are the eigenvectors orthogonal?
 (d) If yes, find the orthonormal eigenvectors.

3. (15%) Use the Stokes' theorem to evaluate

$$\oint_C z^2 e^{x^2} dx + xy^2 dy + \tan^{-1} y dz$$

Where C is the circle given by $x^2 + y^2 = 9$, $z=0$

4. (20%) Using boundary conditions

$$u(0) = 1, \quad v(1) = 0,$$

find the solutions $u(x)$ and $v(x)$, $0 \leq x \leq 1$, to the following systems of ordinary differential equations (ODEs) of (a) and (b)

$$(a) \begin{cases} \frac{du}{dx} = -2u \\ \frac{dv}{dx} = -v \end{cases} \quad (b) \begin{cases} \frac{du}{dx} = u - v \\ \frac{dv}{dx} = v - u \end{cases}$$

5. (30%) Starting each time from initial conditions

$$z(x, 0) = \cos x, \quad -\infty < x < \infty,$$

find the solutions $z(x, t)$ to the following partial differential equations (PDEs) of (a), (b) and (c):

$$(a) \frac{\partial z}{\partial t} + 2 \frac{\partial z}{\partial x} = 0$$

$$(b) \frac{\partial z}{\partial t} + 2 \frac{\partial z}{\partial x} = z$$

$$(c) \frac{\partial z}{\partial t} - 4 \frac{\partial^2 z}{\partial x^2} = 0$$

- (d) Draw your solutions to a)-c) at representative times t_0, t_1, t_2 .