Some round-off constants: $h = 6.63 \times 10^{-34}$ Js; $c = 3.0 \times 10^{8}$ m/s; $\sigma = 5.67 \times 10^{-8}$ W/m$^2$K$^4$; eV $= 1.6 \times 10^{-19}$J; $k_B = 1.38 \times 10^{-23}$JK$^{-1}$ .

[1]The cosmic microwave background (CMB) fits the blackbody radiation spectrum well with a temperature of 2.7 K and a corresponding peak wavelength at 1.9 mm. Applying the relationship between the radiant emittance, i.e. the total power emitted per unit area, and the photon energy density for the blackbody radiation to estimate the number of CMB photons per cm$^3$ in outer space (10%). The high energy cosmic rays are believed to be mostly protons and the maximum energies observed on earth seem to have a bound. This upper bound is explained by collisions between the high energy protons traveling from extra-galactic space and the CMB photons. If the proton energy is high enough and exceeds a threshold value, the following process could occur: $p + \gamma \rightarrow \Delta^+, \Delta^+ \rightarrow p\pi^0$(or $n\pi^+$). Therefore, the proton energy observed on earth is decreased and there exists a so called GZK cut-off. Please determine this energy threshold in eV(10%). The masses of the proton and $\Delta$ baryons are about 938 MeV/$c^2$ and 1232 MeV/$c^2$, respectively.

[2]In 1926, Max Born wrote down a postulate describing the relation between the probability density function and the wave function for a particle moving in one dimension: $P(x,t) = \Psi^*(x,t)\Psi(x,t)$. Use the Schrödinger equation to show that

$$\int_{-\infty}^{\infty} P(x,t)dx$$

is independent of time (10%) and

$$< p >= m\frac{d < x >}{dt} =< \Psi| - i\hbar\frac{\partial}{\partial x}|\Psi >,$$

where $m$ is the mass of this particle (10%).

[3]Describe the phenomena observed and related physical explanations for the following processes: (a) Compton scattering (5%), (b) Raman scattering (5%), (c) Rayleigh scattering (5%), and (d) Rutherford scattering (5%).

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[4]A physical observable $Q$ and the Hamiltonian are represented by the following matrices:

$$Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix};$$

$$H = \begin{pmatrix} 0 & 2i & 0 \\ -2i & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix}.$$

(a) Find the eigen-values and corresponding eigen-vectors for $H$ (10%). (b) If a particle at $t = 0$ is in the state $\Psi(0) = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, find $\Psi(t)$ (5%). (c) Following (b), if you measure $Q$ at time $t$, what is the probability to get $Q = 1$ (5%)?

[5]Explain the Bloch theorem which can be used to describe the electron wave function in a periodic potential (5%). Applying Bloch theorem, one will obtain the band structure or solutions for these electrons in a solid with periodic structure. State explicitly how to use this band-gap idea to distinguish conductor, semiconductor, and insulator (5%). Why there is a need to introduce a positively charged carrier, hole, for semiconductors even though the real physical particle is the negatively charged electron (5%)? What will be the temperature dependence of the electrical conductivity for the typical conductor and semiconductor (5%)? Note that you need to provide the argument.