答案請寫於答卷上

須列計算過程，否則不予計分

總計十題，每題十分

1. Given $m = p/q$ with both p and q are integers, find $\lim\limits_{x \to 1}\left(\dfrac{x^m - 1}{x - 1}\right) = \underline{\hspace{2cm}}$.

2. Given $Y = \dfrac{x_1}{c}\left(b - e^{-c z_1}\right) + a\, e^{-r t_1}\dfrac{x_2}{c}\left(b - e^{-c z_2}\right)$, where a, b, c are constants, Y, x_1, x_2, z_1, z_2 are variables, and $z_i = f_i(x_i)$, $i = 1, 2$, it can be derived that

$$\frac{\partial Y}{\partial x_1} = G(z_1) + H(x_1, Y)\frac{\partial z_1}{\partial x_1}.$$

Please find the expressions of $G(z_1)$ and $H(x_1, Y)$.

3. Find the values of x for which the series
$$1 - 2(x - 1) + 3(x - 1)^2 + \ldots + n(-1)^{n-1}(x - 1)^{n-1} + \ldots$$
converges.

4. If the probability density function (p.d.f.) of a random variable X is as follows, please find the value of c, the mean and the variance of X.

$$f(x) = \begin{cases} -cx & , -1 \le x \le 0 \\ 6cx^3 & , 0 < x \le 1 \end{cases}$$

5. Let $y = (3x - 2)(x - 1)^{-1}$, $z = (y^2 - 1)^{\frac{1}{3}}$, find $\dfrac{dz}{dx} = \underline{\hspace{2cm}}$.

6. If $\lim\limits_{x \to \infty}\left((x^n + a_1 x^{n-1} + a_2 x^{n-2} + \cdots + a_{n-1}x + a_n)^{\frac{1}{n}} - (x + 1)\right) = b$, then $a_1 = \underline{\hspace{2cm}}$.

7. $\displaystyle\int \frac{1}{\sqrt{4x^2 + 4x + 7}}\, dx = \underline{\hspace{2cm}}$.

8. $\displaystyle\int_0^\infty e^{-x}(x^2 + x)\, dx = \underline{\hspace{2cm}}$.

9. Let $f(x, y) = 4x^2 + 4x - 2y^2 + 6$. Find the maximum and minimum values of the function $f(x, y)$ on the region $x^2 + y^2 \le 3$.

10. Let $f(x) = \sqrt{3x^2 + \sqrt{3x^2 + \sqrt{3x^2 + \sqrt{\cdots}}}}$. Find the relation between $f(x)$ and $f'(x)$.