

Instructions:

- Use of any device with computer algebra system during the exam will result in zero points.
- Each answer for problems 1 through 8 need to be clearly labeled with BOTH the problem number and the blank number.
- Show all your work for problems 9 and 10. Make sure to label your final answer.
- Problems 1 through 8 will be graded based on correctness only (5 points each). Problems 9 and 10 (10 points each) will offer partial credits for the work shown.

1. (10%) Evaluate the limits.

$$\bullet \lim_{x \rightarrow 0} \frac{\ln(1 - 3x^2)}{e^{-x} + x - \cos x} = \underline{\hspace{2cm}} \quad (1)$$

$$\bullet \lim_{x \rightarrow \infty} \frac{\ln(1 - 3x^2)}{e^{-x} + x - \cos x} = \underline{\hspace{2cm}} \quad (2)$$

2. (10%) Consider the graph of the function $f(x) = \frac{\sqrt{x^4 + 2x^3} - \sqrt{x^4 - x^3}}{\sqrt{x^2 - 5x}}$.

Find all vertical asymptotes. (3) . (Hint: find the domain)

Find all horizontal asymptotes. (4) .

3. (10%) Consider the curve given by the equation $x^3 + y = 9x\sqrt[3]{y}$.

Find an equation of the tangent line at the point (4, 8). (5) .

Find $\frac{d^2y}{dx^2}$ at the point (4, 8). (6) .

4. (10%) Let f be a smooth function and $F(x) = \int_{\sqrt{2}}^{\sqrt{2x}} \frac{tf(t)}{e^{t^2}} dt$.

Find $F'(x)$. (7) . (Your answer would contain f)

Suppose that $F(x) = f(\sqrt{2x})$. Solve the integral equation for f . (8) .

5. (10%) Let R be the region under $y = \sqrt{x}$, above $y = \ln x$, and between $x = 1$ and $x = 2$.

Find the volume of the solid obtained by rotating R about the x -axis. (9) .

Find the volume of the solid obtained by rotating R about the line $x = 4$. (10) .

6. (10%) Evaluate $\int \left(\frac{1}{x} + \tan^{-1} x - \frac{\pi}{2} \right) dx = \underline{\hspace{2cm}} \quad (11)$.

Determine if the improper integral $\int_1^{\infty} \left(\frac{1}{x} + \tan^{-1} x - \frac{\pi}{2} \right) dx$ is convergent or divergent. Evaluate the improper integral if it is convergent. (12) .

見背面

題號： 23
科目：微積分(C)

題號： 23
共 2 頁之第 2 頁

7. (10%) Evaluate the given double integrals.

$$\bullet \int_0^4 \int_0^{\sqrt{32-2y^2}} y \cos(x^3 - 96x) dx dy = \underline{\hspace{2cm}} \quad (13)$$

$$\bullet \int_0^4 \int_0^{\sqrt{32-2y^2}} \sin(x^2 + 2y^2) dx dy = \underline{\hspace{2cm}} \quad (14)$$

8. (10%) Let $f(x) = e^{x^3} + e^{-x^3}$.

Find the Taylor Series of $f(x)$ at $x = 0$. (15)

Use the Taylor Series to find the value of $f^{(2022)}(0)$. (16)

9. (10%) Sketch the curve $y = (x - 4)\sqrt[3]{x^2}$. Label the following information: (a) Intervals of Increase/Decrease (b) Concavity (c) Local Extrema.

10. (10%) Use the method of Lagrange Multipliers to find the point(s) on the surface $y^2 = 16 + xz$ that are closest to the origin.

試題隨卷繳回