1. **(35 points)** Consider the following linear programming problem.

$$\text{Maximize } z = 4x_1 + 3x_2 + 6x_3$$
$$\text{subject to}$$
$$3x_1 + x_2 + 3x_3 \leq 30$$
$$2x_1 + 2x_2 + 3x_3 \leq 40$$
$$x_1, x_2, x_3 \geq 0$$

(a) (10 points) What are the values of x_2 and x_3 if x_2 and x_3 are the basic variables?

(b) (5 points) The choice of x_1, x_2, x_3 to be the nonbasic variables eliminates the work required to solve for the basic variables x_4 and x_5. Suppose that we choose x_2 to increase from zero. How far can we increase the entering variable x_2 before stopping without leaving the feasible region?

(c) (6 points) Increasing $x_2 = 0$ to its maximum obtained in (b) moves us from the initial basic feasible solution to the new basic feasible solution. What are the values of the new basic variables?

(d) (4 points) What is the improvement in the objective function when we perform the iteration in (c)?

(e) (10 points) Please complete the rest of the simplex method procedures after the initial choice of increasing the entering variable x_2 in (b) and indicate the optimal solutions of x_1, x_2, x_3, and z.

2. **(15 points)** Imagine you have \$10,000 earmarked for investment in the stock market over the next 4 years. The strategy is to purchase the stock at the beginning of each year. The level of risk in the investment is depicted by a probability distribution governing the stock's returns. In each year, there are 3 different market conditions: a triple return with probability 0.4, maintaining the same invested money with probability 0.2, and losing the invested money with probability 0.4. The objective is to formulate an investment policy that maximizes the cumulative money at the end of year 4. Please use dynamic programming to find the investment policy that maximizes the expected amount of money you will have after 4 years.

3. **(15 points)** At the NTU post office, a pair of clerks work at distinct levels of efficiency: clerk 1's service time follows an exponential distribution characterized by the rate μ_1, whereas clerk 2's service time conforms to an alternative exponential distribution with rate μ_2. On a particular day, John arrived at the postal office and started receiving service from clerk 1 at precisely 8:00.

(a) (6 points) Mary enters at 8:10, what is the probability she sees John is still being served by clerk 1?

(b) (9 points) Since John is still in service, Mary goes to clerk 2 to be served. What is the probability that Mary finishes her service before John does?

4. **(10 points)** In the given Markov Chain, assume that the initial state is at state 2 with a probability of 0.2 and at state 4 with a probability of 0.8. What is the probability that, after starting (disregarding the initial state), the process never visits state 2 again?

$$P = \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.8 & 0.2 & 0 \\ 0 & 0 & 1 & 0 \\ 0.2 & 0.3 & 0 & 0.5 \end{bmatrix}.$$

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5. **(25 points)** "The NTU fast-food shop serves two types of customers: ice cream lovers and burger cravers. Both types of customers arrive according to independent Poisson processes with respective rates λ_1 and λ_2. There are two counters in the shop: a general counter and an ice cream counter. The service times at these counters are exponentially distributed with respective rates μ_1 and μ_2. An ice cream lover can be served at both counters but prefers the ice cream counter, whereas burger cravers can only order their meals from the general counter. Due to limited space, queuing inside the shop is not possible. Consequently, both types of customers will leave if they cannot order their desired items immediately upon arrival.

(a) (5 points) Define the necessary states.

(b) (5 points) Formulate the balance equations (no need to solve them).

Assuming that you have solved the balance equations and get the long-run probabilities expressed in algebraic form,

(c) (5 points) what is the average number of customers in the shop?

(d) (5 points) what is the average time a customer spends in the shop?

(e) (5 points) what is the fraction of the general counter's customers that are ice cream lovers?

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