國立臺灣大學 112 學年度碩士班招生考試試題

1. **(15 points)** Consider the following linear programming (LP) model:

$$\begin{aligned} \text{Maximize} \quad & cx \\ \text{Subject to} \quad & Ax \leq b \\ & x \geq 0 \end{aligned}$$

(a) (5 points) Let x_1 and x_2 be feasible points of the LP. Write down the convex combination of x_1 and x_2.

(b) (5 points) Let x' be a point belonging to the convex combination in (a). Show that x' is also feasible.

(c) (5 points) Justify that the objective value of x' is not better than the optimal objective value.

2. **(15 points)** A company has an order from a customer for requiring 9 CNC machines, to be delivered 3 units each month over the next three months. The CNC machines can be shipped to the customer at the end of the same month in which the machines are assembled, or the machines can be stored as inventory for delivery during later months. The inventory holding cost is $400 per machine per month. The company has no current inventory of these CNC machines and desires none after the completion of this contract. The company will determine a production schedule that minimizes the total cost. The production data of assembling the CNC machines are summarized as follows.

	Month 1	Month 2	Month 3
Regular production capacity (units)	2	2	3
Regular production cost per unit	$2500	$2900	$3000
Overtime production capacity (units)	2	2	1
Overtime production cost per unit	$2800	$3400	$3200

(a) (10 points) Construct a **Transportation Tableau** for the problem.

(b) (5 points) Use the **Northwest Corner Method** to find an initial solution.

3. **(20 points)** NTUIIE has four teachers to offer the following courses each semester: Operations Research (OR), Production Planning (PP), Quality Control (QC), and Data Analysis (DA). The effectiveness of each teacher in teaching each class is shown in the following table. Each teacher can teach one course during the semester, and all these four courses are provided.

	OR	PP	QC	DA
Teacher A	9	8	5	5
Teacher B	9	5	6	8
Teacher C	5	8	8	6
Teacher D	7	5	8	9

(a) (10 points) Use the Hungarian Method to solve the teaching assignment problem that maximizes the total effectiveness.

(b) (10 points) Assume that a goal of achieving effectiveness level of about 7 in each course is set by the school, and deviations from this goal in any courses have equal weight. Formulate a goal programming model that determines the teaching assignment.

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4. **(15 points)** A mysterious disease is spreading on a small island, where the population is n. In any single period, two random people from the population meet and interact. If one of the two is infected and the other is not, the disease is transmitted to the healthy person with probability p; otherwise, no disease transmission takes place.

(a) (6 points) Setup a Markov chain model and determine the transition probability matrix.

(b) (9 points) Assuming that the process starts at time 0 with a single person infected, $n = 5$, and $p = 0.1$, what is the mean time that the disease takes to infect the whole population?

(You might need: $\begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & \frac{3}{2} & -\frac{3}{2} & 0 \\ 0 & 0 & \frac{3}{2} & -\frac{3}{2} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{2}{3} & \frac{2}{3} & 1 \\ 0 & \frac{2}{3} & \frac{2}{3} & 1 \\ 0 & 0 & \frac{2}{3} & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$)

5. **(35 points)** The NTU mobile shop is now selling the newest ePhone and managing its own inventory. In the morning of day $(i + 1)$ when the shop opens, it has $S_i + A_i$ ePhones on hands, where S_i is the stock level and A_i is the ordered quantity at the end of day i. The ordered A_i phones will always arrive before the shop opens on day $(i + 1)$. The demand of ePhone on day i, expressed as D_i, occurs during business hours and follows a probability distribution. The inventory control and ordering criteria of the shop are summarized below.

- The shop can only hold 0 to 2 phones at the beginning of the day, i.e., $0 \leq S_i + A_i \leq 2$.
- The shop doesn't allow backorders, i.e., $S_i \geq 0$.
- The penalty cost of unsatisfied demand is 3 per phone.
- Each order issued at the end of the day yields an order handling cost 2 and the variable cost is 4 per phone.
- The day-to-day demands are i.i.d. and follow the probability distribution:

Demand $D_i = d$	0	1	2
Probability $p(D_i = d)$	0.4	0.3	0.3

(a) (5 points) What is the balance equation of the shop regarding $S_i, S_{i+1}, A_i, D_{i+1}$?

(b) (10 points) Let $p_{s_i,s_{i+1}}(a_i)$ indicate the probability from the current (stock level s_i, ordering action a_i) to the next stock level s_{i+1}. Write down the complete transition probability matrix of all possible $p_{s_i,s_{i+1}}(a_i)$.

(c) (10 points) Let $c(s, a)$ denote the incurred cost after taking the ordering action a at the stock level s. List and calculate all the possible costs $c(s, a)$ by varying s and a.

(d) (10 points) The shop currently uses a deterministic ordering policy as shown below. Calculate its expected cost under this policy.

Stock Level s	0	1	2
Ordering Action a	2	0	0