1. (35%) Consider a 2×2 matrix A defined by $A=\begin{pmatrix}1-\alpha & \alpha \\ \beta & 1-\beta\end{pmatrix}$ where $\alpha>0$ and $\beta>0$.
   (a) (5%) Find the determinant of A; .i.e. det A.
   (b) (10%) Under what condition does the inverse of A exist? Find A^{-1} if it exists.
   (c) (10%) Find the eigenvalues of A.
   (d) (10%) Let $\mathbf{x}^{(1)}=\begin{pmatrix}a\\1\end{pmatrix}$ and $\mathbf{x}^{(2)}=\begin{pmatrix}b\\1\end{pmatrix}$ be two independent eigenvectors of A. Determine a and b.

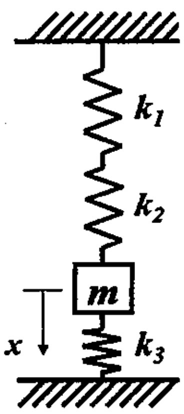
2. (20%) Let $f(x)$ be a function with a period of 2π such that

$$f(x)=\begin{cases}x, & 0<x<\pi\\ \pi, & \pi<x<2\pi\end{cases}$$

   (a) (5%) Is $f(x)$ an even function or odd function?
   (b) (10%) Find the Fourier series of $f(x)$.
   (c) (5%) Based on (b), give an appropriate value to x and show that

$$\frac{\pi}{4}=1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\cdots$$

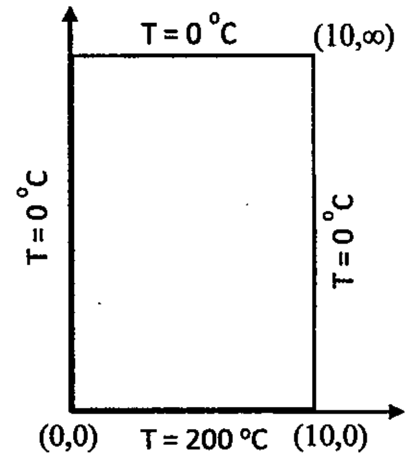
3. (15%) A mass of 5 kg is supported and suspended by three identical springs configured as that shown in the figure to the right. The k_1 and k_2 springs are stretched by a total amount of 0.98 m at the equilibrium position. Assume $x=0$ at the equilibrium position and the acceleration due to gravity is 9.8 m/s^2.
   (a) (5%) Find the spring constant of each spring.
   (b) (5%) Find the undamped natural frequency of the system.
   (c) (5%) Now, suppose that the mass is released at the position with the natural length of the springs when $t=0$ (i.e., $x(0)=-0.98$ m) with zero initial velocity and that the damping due to air resistance is 35 N·s/m. Find the motion as a function of time after release, i.e., $x(t)$.



4. (30%) Consider a semi-infinite one-dimensional plate shown in the right figure. At $y=0$, the temperature is maintained at 200 °C. All the other three sides have temperatures at 0 °C. The following partial differential equation can describe the temperature distribution of this plate, and its solution can be written as $T(x,y)=X(x)Y(y)$.

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0$$

   (a) (5%) What is the type of this PDE?
   (b) (5%) What is the order of this PDE?
   (c) (10%) Solve the general solutions of $X(x)$ and $Y(y)$, including the separation constant.
   (d) (10%) Solve the steady-state solution using the boundary conditions.



試題隨卷繳回