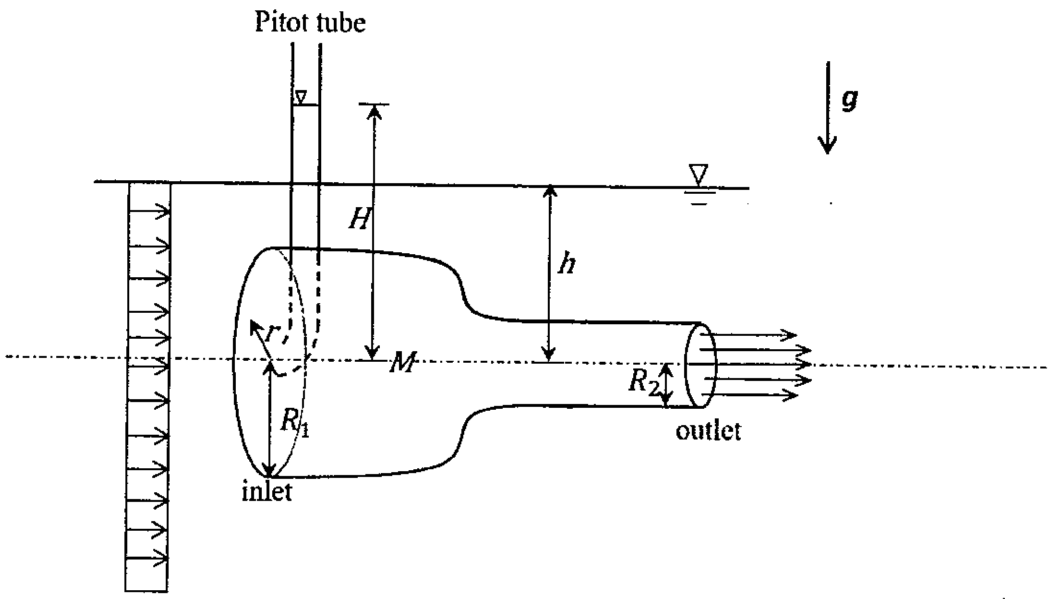
1. As shown in the figure, a funnel (with the circular cross-section and a smooth wall) is placed in the water, of density ρ, at a depth h. The funnel does not move. A Pitot tube is placed at the inlet of the funnel to measure the velocity of the water flow. Inside the Pitot tube, the height of the water column is H. The radius of the cross-section area at the inlet of the funnel is R_1 and at the outlet is R_2.
   (1) (5 pts) What is the flow velocity at the inlet?
   (2) (5 pts) If you move the Pitot tube toward the outlet (e.g., move it to the point M in the figure), will the water level in the Pitot tube retain the same, decrease, or increase? State the reason in detail.
   (3) (5 pts) At the outlet of the funnel, for the slow flow case where the viscosity dominates, the velocity distribution can be described as

$$u_2(r) = \frac{C}{\nu}(R_2^2 - r^2),$$

   where ν is the kinematic viscosity of the water, r is the coordinate in the radial direction, and C is a (dimensional) coefficient. Find the value of C.
   (4) (10 pts) Now, in the fast flow case, assuming that the velocity at the outlet is uniform by neglecting viscosity ($u_2 \approx$ constant), what is the force required to hold the funnel in place?



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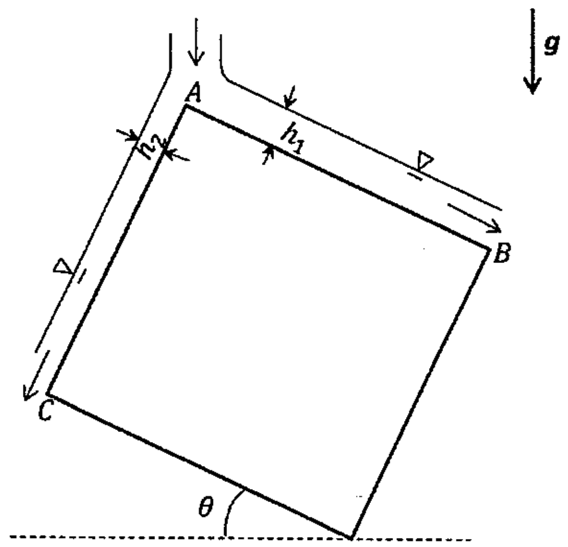
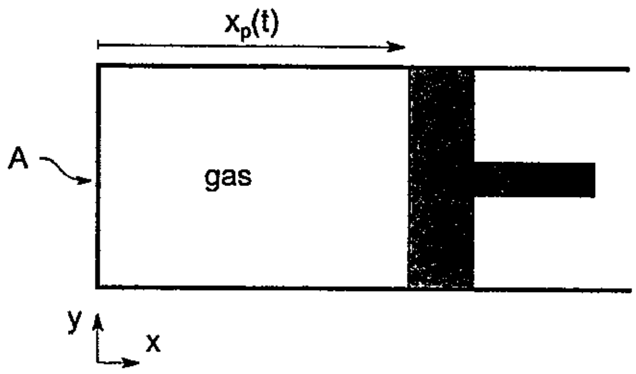
2. As shown in the figure, a stream of viscous liquid, of viscosity μ and density ρ, discharged from a slot onto the upper surface of a brick, forming two branches of the liquid film on the brick's surface $\overline{AB}$ and $\overline{AC}$. The brick is inclined at an angle θ with the horizontal plane. The measured depths of the liquid film are h_1 on $\overline{AB}$ and h_2 on $\overline{AC}$.
(1) (5 pts) Sketch the velocity profiles.
(2) (15 pts) Solve for the velocity profiles of the liquid film on the brick's surface. State all assumptions you have to make.
(3) (5 pts) If the flow rate on the brick's surface is uniform, find the ratio of the film's depths, h_1/h_2.



3. A piston moves slowly inside a cylinder closed at one end, as shown in the figure, and its position $x_p(t)$ is a prescribed function of time t. The initial position of the piston is x_0, the initial gas density is ρ_0, and the area of the cylinder is A. Please find the gas density ρ and the gas velocity $\vec{u}$ for the following conditions. Since the motion is slow, the gas density ρ is assumed to be uniform within the cylinder but varies with time. (Hint: mass conservation).
(a) (12.5 pts) The position of the piston is $x_p(t) = x_0 + v_0 t$. Here the moving velocity v_0 is a constant.
(b) (12.5 pts) The position of the piston is $x_p(t) = x_0 + a_0 \sin(\omega_0 t)$. Here the amplitude of the oscillation a_0 and the angular velocity ω_0 are constants.



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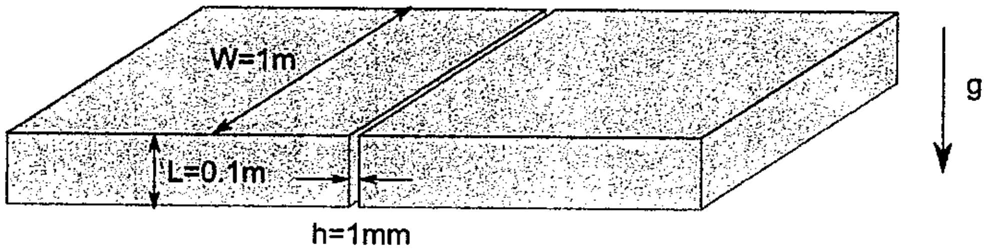
4. (25 pts) A flat roof of a building is constructed of precast concrete slabs of width $W = 1\text{m}$ and depth $L = 0.1\text{m}$, but the end joint between two slabs is not sealed, leaving a crack of width $h = 1\text{mm}$, as shown in the figure. As it rains, the rainwater drips into the interior of the building through the crack. To catch the incoming water, we place a 100-Liter bucket under the leak. Please find out how long it would take to fill the bucket. The roof is 3 meters above the ground, the dynamic viscosity of the water $\mu = 1.13 \times 10^{-3}\text{Pa}\cdot\text{s}$, the mass density of the water $\rho = 1000\text{kg/m}^3$, and the gravitation acceleration $g = 9.8\text{m/s}^2$. The rainwater passing through the crack is assumed to be a steady laminar viscous flow, and the influence of the surface tension is neglected.



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