

1.(25%) Solve the following differential equation.

$$y' = \frac{20(x \cos y - e^y)}{x(4x \sin y + 5e^y)}$$

2.(25%) Find the general solution of

$$y'' + 2y' + 2y = e^{-t} \tan(t), \quad -\frac{\pi}{2} < t < \frac{\pi}{2}.$$

3.(25%) (a.) Find the general solution of the following system of equations.

$$\begin{cases} x' = 5x + y - 4z, \\ y' = 3x + 5y - 7z, \\ z' = 2x + y - z. \end{cases}$$

(b.) Find all the equilibrium solutions. Determine whether they are stable, asymptotically stable, or unstable.

4.(25%) We consider the differential equation

$$y''' + p(t)y'' + q(t)y' + r(t)y = 0, \quad (1)$$

where  $p$ ,  $q$ , and  $r$  are continuous functions on an open interval  $I$ . Let  $y_1$ ,  $y_2$ , and  $y_3$  be solutions of above equation (1). Show that the Wronskian of  $y_1$ ,  $y_2$ , and  $y_3$  are either identical zero or never zero on  $I$ .

試題隨卷繳回