1. **(10 points)** Consider the following linear programming (LP) model and its optimal solution:

$$\max z = 6x_1 + 10x_2 + 9x_3 + 20x_4$$

subject to

$$4x_1 + 9x_2 + 7x_3 + 10x_4 \le 600$$
$$x_1 + x_2 + 3x_3 + 40x_4 \le 400$$
$$3x_1 + 4x_2 + 2x_3 + x_4 \le 500$$
$$x_1, x_2, x_3, x_4 \ge 0$$

The optimal solution: $z = \frac{2800}{3}$, $(x_1, x_2, x_3, x_4) = (\frac{400}{3}, 0, 0, \frac{20}{3})$, and slack variables $(s_1, s_2, s_3) = (0, 0, \frac{280}{3})$.

(a) (5 points) Formulate its dual problem.

(b) (5 points) Find the optimal solution to the dual by using the given optimal primal solution and the Theorem of Complementary Slackness.

2. **(20 points)** Two students (say, A and B) form a group to conduct their final project for the course of Operations Research. After defining the project topic, they divide the project into four tasks: model formulation, implementation, numerical study, and reporting, and each student will be assigned two tasks. Their efficiencies on these tasks differ, where the time each would need to perform the task is shown in the following table.

	Time required on each task (hours)			
	Formulation	Implementation	Numerical	Reporting
Student A	5	7.3	3.2	4.6
Student B	5.4	6.7	3.8	4.9

(a) (5 points) Formulate the problem as a transportation problem where the objective is to minimize the total time spent on the project.

(b) (10 points) Formulate the problem where the objective is to minimize the time spent for each student.

(c) (5 points) If the above problems are optimally solved, which problem (problem a or problem b) will yield the minimal amount of time spent on the entire project. Justify your answer.

3. **(20 points)** *NIE* is a small but growing manufacturing company, and *NIE* is planning to install a mechanized conveyer system to transport finish goods from the production line to its warehouse in 3 years. Before that, *NIE* is currently purchasing a new cargo tractor to perform the task. The running cost of the tractor will increase rapidly as the tractor ages, and thus it may be more economical to replace the tractor after 1 or 2 years. The following table give the total net discounted cost associated with purchasing a new tractor (purchase price minus the trade-in allowance, and plus running cost) at the end of year i and trading it in at the end of year j (where year 0 is now).

	j		
	1	2	3
i=0	$10000	$20000	$33000
i=1		$12000	$23000
i=2			$14000

(a) (10 points) What network optimization model can be used to solve this problem so that the total cost over 3 years can be minimized? Also draw the corresponding network graph.

(b) (10 points) Based on (a), formulate the corresponding linear programming model.

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4. **(20 points)** NTUShop has three grocery stores on campus. The NTUShop has purchased five boxes of ice cream. The stores adopt the policy of price discrimination where various stores may determine different prices of ice cream; therefore, the profit generated by a store may be different from other stores. The NTUShop would like to know how to allocate five boxes of ice cream to the three stores on campus to maximize the total profit. The following table gives the estimated profit at each store when it is allocated various number of boxes of ice cream. It is not allowed to split a box of ice cream between stores. However, it is allowed to allocate no boxes of ice cream to any of the NTUShop stores.

	NTUShop Store		
Boxes of ice cream	1	2	3
0	0	0	0
1	5	6	4
2	9	11	9
3	14	15	13
4	17	19	18
5	21	22	20

Please use dynamic programming to determine how many of the five boxes of ice cream should be assigned to each of the three NTUShop stores to maximize the total profit.

5. **(15 points)** A food delivery company, Foodbear, has divided the city into three areas – Downtown, the North End and the South End. A food delivery driver starting his ride in the North End: 50% stays in the region, 20% goes to the Downtown, and 30% goes to the South End. A food delivery driver starting his ride in the Downtown: 10% goes to the North End, 40% stays in the Downtown, and 50% goes to the South End. A food delivery driver starting his ride in the South End: 30% goes to the North End, 30% goes to the Downtown, and 40% stays in the South End.

(a) (5 points) What is the transition matrix?

(b) (5 points) If a food delivery driver starts his work in the Downtown, what is the probability that he will be in the Downtown after completing his 3rd ride?

(c) (5 points) The food delivery driver noticed that the fees earned in the Downtown yield on average \$100, in the South End \$120, and in the North End \$150. He also noticed that on average he has 15 rides per day. What is his expected total earning in a month (30 days)?

6. **(15 points)** Consider the following unconstrained nonlinear program with two variables.

$$f(x_1, x_2) = (x_1 - 1)^2 + x_2^2 + 1$$

(a) (4 points) What is the gradient of this problem?

(b) (4 points) What is the Hessian matrix of this problem?

(c) (4 points) Check the definiteness of the Hessian matrix of this problem.

(d) (3 points) Solve this nonlinear program.

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