

- The rigid bars  $AB$  and  $BC$  of Fig. 1 are connected at Point  $B$  using a hinge connection and supported using a hinge, a linear elastic spring with a spring constant of  $\beta$ , and a roller at Points  $A$ ,  $B$ , and  $C$ , respectively, as shown in Fig. 1. The spring constants of the two rotational springs at Points  $A$  and  $B$  are both  $\beta_R$ . The lengths of Bars  $AB$  and  $BC$  are both  $L/2$ . The structure is subjected to a vertical axial load  $P$ . Find the critical axial load  $P_{cr}$  of the structure. (20%)
- The cantilever beam of Fig. 2-1 has a total length of 4 meters and is loaded by a uniformly distributed load with an intensity of  $q$  N/m. The beam is supported at the mid-span point  $B$  by a linear rotational spring with a spring constant  $k_R$  of  $5 \times 10^6$  N-m, which can provide a resisting moment  $M_B = k_R \times \theta_B$  when subjected to a rotational angle of  $\theta_B$ . The cross section of the beam is shown in Fig. 2-2 and composed of a wood section of  $150 \text{ mm} \times 250 \text{ mm}$  and two brass plates both with a cross section of  $150 \text{ mm} \times 50 \text{ mm}$ . The two brass plates are at the top and bottom of the beam. The moduli of elasticity  $E$  of the wood and brass in the beam are 15 and 100 GPa, respectively. Ignore the self-weight of the beam. (30%)
  - Determine the magnitudes and directions of the reactions at Points  $A$  and  $B$ , namely,  $M_A$ ,  $R_A$ , and  $M_B$ . Express the answers as a function of  $q$ .
  - Assume the allowable normal stresses  $\sigma$  of the wood and brass in the beam are 10 and 70 MPa, respectively. Find the maximum value of  $q$  so that the stresses in the beam are all allowable.

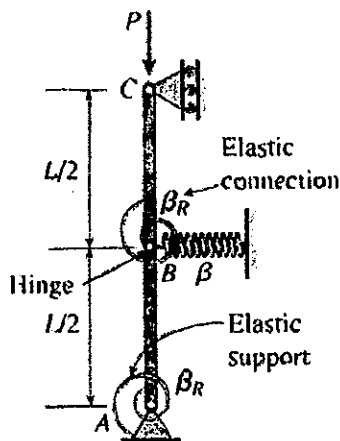


Fig. 1

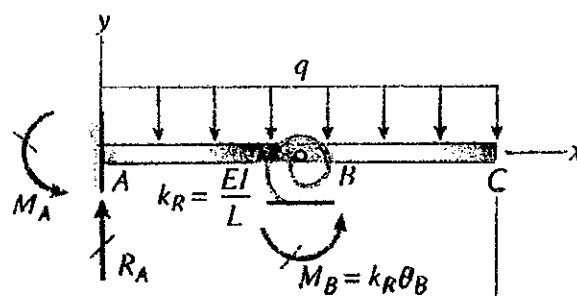


Fig. 2-1

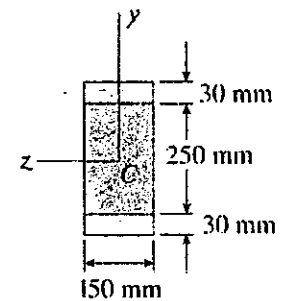


Fig. 2-2

Problems 3 and 4 are in the next page.

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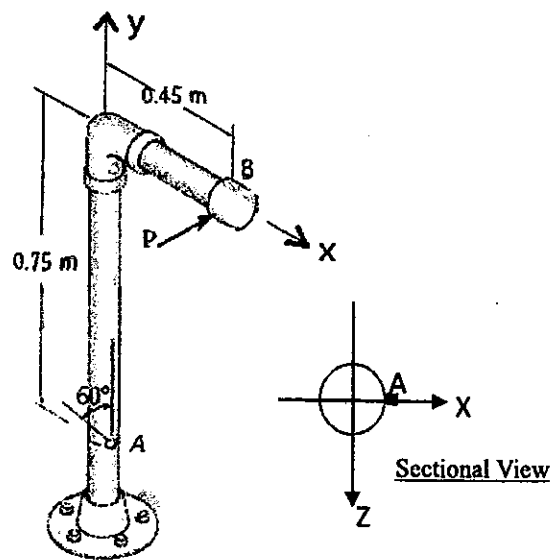
3. The L-shape pipe of Fig. 3 is placed in the vertical x-y plane and subjected to a horizontal force  $P$  at Point  $B$  in the  $-z$  direction. A single strain gauge is attached at Point  $A$ , which is on the outer surface of the pipe (see Fig. 3). The strain gauge is attached at an angle of  $60^\circ$  to the longitudinal direction of the pipe and gives a reading of  $-100 \times 10^{-6}$ . The pipe has an outer diameter of 60 mm and an inner diameter of 30mm and made of a material with a modulus of elasticity  $E$  of 200 GPa and a shear modulus of elasticity  $G$  of 75 GPa. Determine (1) the horizontal force  $P$ , and (2) the principal stresses at Point  $A$ .

(25%)

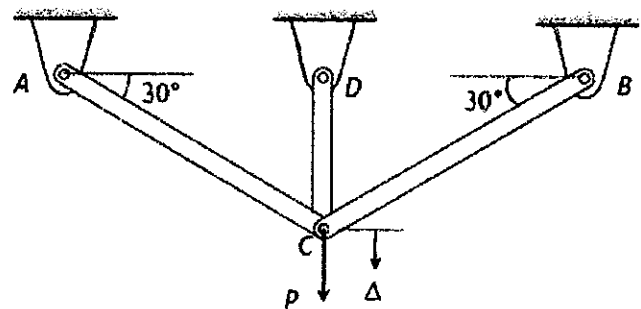
4. The symmetric three-rod truss of Fig. 4 is subjected to a force  $P$  at Point  $C$ . The three circular rods have the same cross sectional area of  $10 \text{ cm}^2$  and are all made of elastic-perfectly plastic steel with a modulus of elasticity of 200 GPa and a yielding stress of 200 MPa. The length of Rods  $AC$  and  $BC$  are both 2 meters.

(25%)

- (1) If  $P = 200 \text{ kN}$ , what is the displacement at Point  $C$ ?  
 (2) If  $P = 300 \text{ kN}$ , what is the displacement at Point  $C$ ?  
 (3) If  $P$  is applied to the truss with its magnitude slowly increased from 0 to 300 kN, and then unloaded from 300 kN to 0, what is the residual displacement at Point  $C$  and what are the residual forces in Rods  $AC$  and  $DC$ ?



**Fig. 3**



**Fig. 4**

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