1. (30%) For a matrix $\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

   (a) (3%) What is the rank of $\mathbf{A}$?

   (b) (3%) Compute the determinant of $\mathbf{A}$

   (c) (3%) Does $\mathbf{A}^{-1}$ exist? If so, find $\mathbf{A}^{-1}$

   (d) (9%) Solve the eigenvalues and eigenvectors of $\mathbf{A}$

   (e) (6%) Diagonalize $\mathbf{A}$ if possible

   (f) (6%) Compute $\mathbf{A}^{20}$

2. (30%) Suppose $F(x, y, z) = (0, -z^2, yz)$

   (a) (5%) Does there exist a vector field G such that $\nabla \times G = F$? Explain why or why not

   (b) (5%) Is F a gradient vector field? Justify your answer

   (c) (10%) Let $S = \{(x, y, z): x^2 + y^2 + z^2 = 1, x \geq 0\}$ Compute the surface integral $\iint_S (\nabla \times F) \cdot dS$ without using

   any integral theorems

   (d) (10%) Verify your answer in (c) by using Stokes' theorem

3. (40%) For $y'' + 4y = g(t)$, $y(0) = 0$ and $y'(0) = 0$

   (a) (20%) Solve $y(t)$ for $g(t) = \sin^2 t$

   (b) (20%) Solve $y(t)$ for $g(t) = \begin{cases} 0 & 0 \leq t < 1 \\ t - 1 & 1 \leq t < 2 \\ 1 & t \geq 2 \end{cases}$

試題隨卷繳回