1. (15 points) The following is an incorrect argument. Identify three mistakes. Briefly explain your answer.

| | | |
|---|---|---|
| 1 | $\exists x\, P(x) \wedge \exists x\, Q(x)$ | Premise |
| 2 | $\forall x\, (R(x) \rightarrow (P(x) \wedge Q(x)))$ | Premise |
| 3 | $P(c) \wedge Q(c)$ | Existential Instantiation 1 |
| 4 | $R(c) \rightarrow (P(c) \wedge Q(c))$ | Universal Instantiation 2 |
| 5 | $R(c)$ | Modus Tollens 3,4 |
| 6 | $\forall x\, R(x)$ | Universal generalization 5 |

Conclusion:

"$\forall x\, R(x)$"

2. (15 points) Compute

$$\sum_{k=0}^{\infty}(2k+1)\cdot\frac{1}{3^{2k}}$$

3. (10 points) Solve the following recurrence:

$$a_1 = 3, \tag{1}$$
$$a_n = a_{n-1}^2 - 2a_{n-1} + 2, \text{ for all } n \geq 2. \tag{2}$$

4. (15 points) Let $S$ be the set of all functions that have domain $\{0,1\}$ and codomain $\mathbb{N}$ (i.e. maps the set $\{0,1\}$ to $\mathbb{N}$). Is $S$ countable? Prove your answer.

5. (15 points) Let $R$ be the relation $\{(a,b)|\, b-a=2\}$ on the set $\{2,3,4,5,7,8\}$.

   (a) Find the transitive closure of $R$.

   (b) Find all maximal elements.

   (c) Find two different total orderings that are compatible with relation $R$. You may describe the total orderings using Hasse diagram for simplicity.

6. (15 points) A *cut edge* is an edge whose removal disconnects the graph. Prove or disprove the following:

   (a) If $G$ is a connected graph in which the degree of every vertex is even. $G$ does not have cut edges.

   (b) If $G$ is a connected graph in which the degree of every vertex is a multiple of three, then $G$ does not have cut edges.

7. (15 points) Let $S$ be a set of $n$ points in the plane such that the distance between any two points is at least one. Prove that there are at most $3n$ pairs of points with distance exactly one.

試題隨卷繳回