1. (7%) Let A be an $m \times n$ matrix, and let E_1 and E_2 be an $m \times m$ elementary matrix and an $n \times n$ elementary matrix, respectively. Which of the following statements are true?

(A) Row A = Row E_1A.
(B) Col A = Col AE_2.
(C) Null A = Null AE_2.
(D) Rank A = Rank E_1A.
(E) None of the above statements is true.

2. (7%) Consider an $m \times m$ matrix P, an $n \times n$ matrix Q, an $m \times n$ matrix A, and an $n \times p$ matrix B. Which of the following statements are true?

(A) Null A is a subspace of Null PA.
(B) Null PA is a subspace of Null A.
(C) Null AQ may not be a subspace of Null A and Null A may not be a subspace of Null AQ.
(D) Col A is contained in Col AB.
(E) The nullity of B is no greater than the nullity of AB.

3. (7%) Let A and B be $m \times n$ matrices. Which of the following statements are true?

(A) Rank A + dim (Null A^T) = n.
(B) $(A+B)\mathbf{x} = \mathbf{b}$ is consistent for every $\mathbf{b}$ in $\mathbb{R}^m$ if and only if the rank of $A+B$ is n.
(C) Every column of the matrix B is a linear combination of its pivot columns.
(D) Suppose A is a standard matrix of a linear transformation T. If the rank of A is m, then T is one-to-one.
(E) If A and B are invertible, then AB is invertible.

4. (7%) Let $A = \begin{bmatrix} 1 & 2 & -1 & 2 & 1 \\ -1 & -2 & 1 & 2 & 3 \\ 2 & 4 & -3 & 2 & 0 \\ -3 & -6 & 2 & 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 & 1 & 1 \\ -1 & -2 & 2 & 1 & 3 \\ 2 & 4 & -4 & 1 & 0 \\ -3 & -6 & 6 & 0 & 3 \\ 1 & -2 & 2 & 5 & 3 \end{bmatrix}$.

Which of the following statements are true?

(A) The rank of A = 2.
(B) The rank of B = 3.
(C) The dimension of null space of A = 3.
(D) B is an invertible matrix.
(E) The third column of AB is a pivot column.

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5. (7%) Suppose $T:\mathbb{R}^3 \to \mathbb{R}^3$ has two-dimensional range and we know

$$T\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right)=\begin{bmatrix}1\\2\\-1\end{bmatrix} \text{ and } T\left(\begin{bmatrix}1\\1\\1\end{bmatrix}\right)=\begin{bmatrix}0\\0\\0\end{bmatrix}.$$

Which of the following are a possible standard matrix of T?

(A) $\begin{bmatrix}1 & 1 & -2\\0 & 2 & -2\\0 & -1 & 1\end{bmatrix}$

(B) $\begin{bmatrix}-1 & 1 & 0\\-2 & 2 & 0\\1 & -1 & 0\end{bmatrix}$

(C) $\begin{bmatrix}2 & -1 & -1\\2 & -2 & 0\\-1 & 1 & 0\end{bmatrix}$

(D) $\begin{bmatrix}1 & 1 & -2\\2 & 2 & -4\\1 & -1 & -2\end{bmatrix}$

(E) $\begin{bmatrix}-1 & 1 & 0\\0 & 2 & -2\\0 & -1 & 1\end{bmatrix}$

6. (7%) Which of the following statements are true?

(A) If C is a matrix whose columns form a basis for a subspace W which is $\mathbb{R}^n$, then $C(C^TC)^{-1}C^T = I_n$.
(B) A diagonalizable matrix is *similar* to a diagonal matrix.
(C) If F and G are subsets of $\mathbb{R}^n$ and $F^\perp = G^\perp$, then $F = G$.
(D) If P is a matrix whose columns are eigenvectors of a symmetric matrix, then P is orthogonal.
(E) Distinct eigenvectors of a symmetric matrix are orthogonal.

7. (8%) Consider the vector space of all polynomials in x:

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$$\mathcal{P} = \left\{ \sum_{k=0}^{n} a_k x^k \mid n \in \mathbb{Z}, n \geq 0, a_k \in \mathbb{R} \right\}$$

with the vector addition and scalar multiplication defined as the regular polynomial addition and scalar multiplication. Which of the following are an inner product with $p(x) = \sum_{k=0}^{n_p} p_k x^k$ and $q(x) = \sum_{k=0}^{n_q} q_k x^k$.

(A) $\langle p, q \rangle = \frac{1}{2}\int_{-2}^{2} p(x)q(x)dx$.

(B) $\langle p, q \rangle = \int_{-1}^{3} p(x)q(x)dx$.

(C) $\langle p, q \rangle = \sum_{k=0}^{N} p_k q_k$, where $N = \min\{n_p, n_q\}$.

(D) $\langle p, q \rangle = \sum_{k=0}^{N} k \cdot p_k q_k$, where $N = \min\{n_p, n_q\}$.

(E) $\langle p, q \rangle = \sum_{k=0}^{N} (k+1) p_k q_k$, where $N = \min\{n_p, n_q\}$.

8. (5%) Which of the following differential equations are exact?

(A) $(5x + 4y)dx + (4x - 8y^3)dy = 0$
(B) $(x^2 - y^2)dx + (x^2 - 2xy)dy = 0$
(C) $(y \ln y - e^{-xy})dx + (\frac{1}{y} + x \ln y)dy = 0$
(D) $(\tan x - \sin x \sin y)dx + (\cos x \cos y)dy = 0$
(E) none of the above

9. (5%) Find the solution of the differential equation $(y^2 + xy^3)dx + (5y^2 - xy + y^3 \sin y)dy = 0$.

(A) $xy + \frac{1}{4}x^2 + 2\ln|x| - \cos x = c$; c is a constant.

(B) $xy + \frac{1}{4}x^2 + 5\ln|x| - \cos x = c$; c is a constant.

(C) $\frac{x}{y} + \frac{1}{2}x^2 + 5\ln|y| - \cos y = c$; c is a constant.

(D) $\frac{2x}{y} + \frac{1}{4}x^2 + 2\ln|y| - \cos y = c$; c is a constant.

(E) none of the above

10. (5%) Which of the following functions y=f(x) could be annihilated by the given differential operator

$D^2(D^2-6D+10)(D-5)(D-7)$, where $D^k y=\dfrac{d^k y}{dx^k}$ and k could be 0,1,2,3…?

(A) $e^{2x}\cos x$
(B) $e^{3x}\cos 2x$
(C) $e^{3x}\sin x$
(D) e^{5x}
(E) x

11. (5%) The particular solution of the differential equation $y^{(4)}-4y''=5x^2-e^{2x}$ is $y_p=Ax^2+Bx^3+Cx^4+Dxe^{2x}$. Which of the following is the value of A+B+C+D?

(A) $-\dfrac{23}{48}$
(B) $-\dfrac{21}{48}$
(C) $\dfrac{21}{48}$
(D) $\dfrac{21}{47}$
(E) $-\dfrac{21}{47}$

12. (5%) Which of the following is the particular solution of the differential equation $y''-2y'+y=e^t\arctan t$?

(A) $\frac{1}{4}e^t[(t^2-1)\cot(t)-\ln(2+t^2)]$
(B) $\frac{1}{4}e^t[(t^2-1)\tan(t)-\ln(2+t^2)]$
(C) $\frac{1}{4}e^{2t}[(t^2-1)\tan^{-1}(t)-\ln(1+t^2)]$
(D) $\frac{1}{2}e^t[(t^2-1)\cot^{-1}(t)-\ln(2+t^2)]$
(E) $\frac{1}{2}e^t[(t^2-1)\tan^{-1}(t)-\ln(1+t^2)]$

13. (5%) Which of the following sentences are correct about the differential equation $y''-\dfrac{1}{x}y'+\dfrac{1}{(x-1)^3}y=0$?

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(A) This is nonlinear ordinary differential equation.

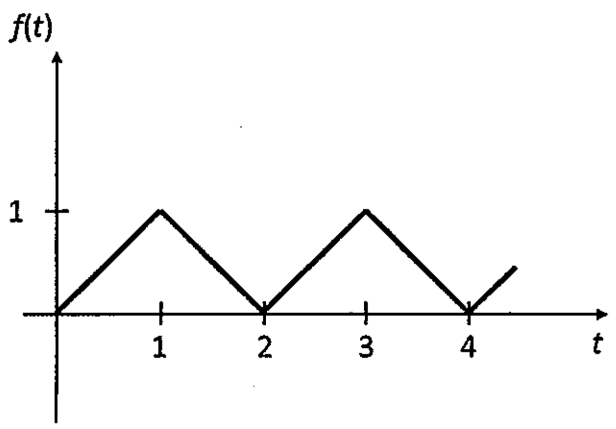
(B) Regular singular point: $x=0$

(C) Regular singular point: $x=1$

(D) Irregular singular point: $x=0$

(E) Irregular singular point: $x=1$

14. (5%) Find the Laplace transform of the given periodic function (triangular wave).



(A) $\dfrac{1-e^{-2s}}{s\ (1+e^{-2s})}$

(B) $\dfrac{5-e^{-3s}}{s^2(1-e^{-2s})}$

(C) $\dfrac{1-e^{-2s}}{s^3(1-e^{-2s})}$

(D) $\dfrac{1-e^{-s}}{s^2(1-e^{-2s})}$

(E) $\dfrac{2-e^{-s}}{s\ (3-e^{-2s})}$

15. (5%) Solve the following equation $f(t)+2\int_0^t f(\tau)\cos(t-\tau)d\tau = 4e^{-t}+\sin t$ Which of the following is $f(t)$?

(A) $4e^{-t}-9te^{-t}+2t^3e^{-t}$

(B) $4e^{-t}-9t^2e^{-t}+4t^2e^{-t}$

(C) $4e^{-t}-9te^{-t}+3t^2e^{-t}$

(D) $4e^{-2t}+7te^{-t}+4t\ e^{-t}$

(E) $4e^{-t}-7te^{-t}+4t^2e^{-t}$

16.(5%) Find the general solution $X=\begin{pmatrix}x(t)\\y(t)\\z(t)\end{pmatrix}$ of given differential equations $\begin{matrix}\frac{dx}{dt}=3x+2y+4z\\ \frac{dy}{dt}=2x+2z\\ \frac{dz}{dt}=4x+2y+3z\end{matrix}$.

(A) $X=c_1\begin{pmatrix}2\\1\\2\end{pmatrix}e^{t}+c_2\begin{pmatrix}0\\-2\\1\end{pmatrix}e^{-8t}+c_3\begin{pmatrix}1\\1\\0\end{pmatrix}e^{-8t}$

(B) $X=c_1\begin{pmatrix}2\\1\\2\end{pmatrix}e^{8t}+c_2\begin{pmatrix}0\\-3\\2\end{pmatrix}e^{-t}+c_3\begin{pmatrix}1\\1\\0\end{pmatrix}e^{-t}$

(C) $X=c_1\begin{pmatrix}2\\1\\2\end{pmatrix}e^{8t}+c_2\begin{pmatrix}0\\-2\\1\end{pmatrix}e^{-t}+c_3\begin{pmatrix}1\\-2\\0\end{pmatrix}e^{-t}$

(D) $X=c_1\begin{pmatrix}2\\1\\2\end{pmatrix}e^{t}+c_2\begin{pmatrix}0\\-2\\2\end{pmatrix}e^{-6t}+c_3\begin{pmatrix}1\\1\\0\end{pmatrix}e^{-2t}$

(E) none of the above

17. (5%) Find the Fourier transform of xe^{-x^2}.

(A) $-\frac{i\omega}{2}\sqrt{\frac{\pi}{2}}e^{-\frac{\omega^2}{4}}$

(B) $-\frac{i\omega}{2}\sqrt{\pi}e^{-\frac{\omega^2}{4}}$

(C) $\frac{i\omega}{2}\sqrt{\pi}e^{-\frac{\omega^2}{4}}$

(D) $\frac{i\omega}{2}\sqrt{\frac{\pi}{2}}e^{-\frac{\omega^2}{4}}$

(E) none of the above

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