1. **[10%] Majorana particle** In the field of particle physics, every type of particle (粒子) has an associated antiparticle (反粒子) with the same mass but with opposite physical charges (such as electric charge). For instance, the antiparticle of the electron (電子) is the positron (正子). While the electron has a negative electric charge, the positron has a positive electric charge.
A Majorana particle is a particle that is its own antiparticle. This was hypothesized by Ettore Majorana in 1937. What is the charge of a Majorana particle? (5 %) Why? (5 %)

2. **[20%] Classical Hall effect, integer quantum Hall effect, fractional quantum Hall effect and fractionally-charged quasi-particles in two dimensions** Please prove that in a two-dimensional (2D) electron system, the Hall resistance (霍爾電阻) is given by $R_{\mathrm{H}} \equiv \frac{V_{\mathrm{H}}}{I} = \frac{B}{ne}$, where V_{H} is the Hall potential difference, I is the current, B is the applied magnetic field perpendicular to the 2D electron system, n is the carrier concentration (in m^{-2}) and e is the electron charge, respectively. (5 %) [In two dimensions, the current density is given by $J = \frac{I}{w}$ where w is the width of the 2D system. Ohm's law is given by $E = \rho_{\mathrm{H}} J$ where $R_H = \rho_{\mathrm{H}}$ (the Hall resistivity in two dimensions)]. In the low-temperature, high magnetic field (低溫高磁) regime, the Landau levels can form and whose energy are given by $(n+\frac{1}{2})\hbar\omega_{\mathrm{C}}$, where $n = 0, 1, 2, 3$....... Here $\omega_{\mathrm{C}} = \frac{eB}{m^*}$, m^* is the effective mass of an electron in the system and $\hbar = \frac{h}{2\pi}$ is the reduced Planck constant. Given the 2D density of states (the number of allowable states per unit energy per unit area) is $\frac{m^*}{(\pi)(\hbar^2)}$ (we have already considered spin degeneracy), please prove the Landau level degeneracy is given by $\frac{eB}{h}$. (5 %) The Landau level filling factor (the number of Landau levels below the Fermi energy) is given by $\nu = \frac{nh}{eB}$, please prove that $R_{\mathrm{H}} = \frac{h}{(\nu)(e^2)}$ (5 %). Here ν is an integer, and the Hall resistance is quantized and thus is called the integer quantum Hall effect. The fractional quantum Hall effect was first observed when $R_{\mathrm{H}} = \frac{h}{(1/3)e^2} = \frac{3h}{e^2}$. Please prove that in the mean-field approximation, at $\nu = 1/3$ three magnetic flux quanta [each in $(\frac{h}{e})$] are bound to one electron. (平均而言，一個電子配上 3 個磁通量量子)(5 %). Finally, if we consider the quasi-hole picture in which an electron is removed, the three magnetic flux quanta are no longer bound to an electron. Effectively a fractionally charged quasi-hole with change $e/3$ can be observed.

3. **[10%] Single-photon interference** Young's two-slit experiment, one photon at a time (單光子楊式雙狹縫干涉實驗). Shall we still see the classic interference pattern if the light level is so low that there is only one photon in the apparatus at a time? (Please answer yes or no) (5 %)? Why? (5 %)

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4. **[10%] *p*-wave superconductivity** The conventional Bardeen-Cooper-Schrieffer (BCS) theory of superconductivity showed that the condensed state consists of bound electron pairs of opposite spin (spin-singlet) [自旋反向的電子對]. In contrast, a *p*-wave superconductor, is a spin-triplet [自旋同向的電子對] superconductor. In your own words, what can be considered as experimental evidence for a *p*-wave superconductor? (10%)

5. **[20%] Basic principles in modern physics** Quantum phenomena in modern physics are often negligible in the "macroscopic" world. In the following cases show the fact *numerically*.

(1) (10%) The amplitude of the zero-point oscillation for a pendulum of length $L = 1$m and mass $m = 1$kg.

(2) (10%) The diffraction angle of a tennis ball of mass $m = 0.1$ kg moving at a speed $v = 10$ m/s by a window of size 1.0×1.0 m^2.

6. **[15%] Stern-Gerlach experiment** Considering an experiment in which a beam of electrons is directed at a plate containing two slits, labeled **A** and **B**. In addition, beyond the plate is a screen to detect the bam intensity hitting the screen. For each of the following cases draw a rough graph indicating the beam intensity detected at the screen and give a brief explanation, respectively.

(1) (5%) Stern-Geralch apparatus attached to the slits. Therefore, only electrons with $s_z = h/2$ can pass through slit **A** and only electrons with $s_z = -h/2$ can pass through **B**.

(2) (5%) Only electrons with $s_z = h/2$ can pass through slit **A** and only electrons with $s_z = h/2$ can pass through **B**.

(3) (5%) Both slits open for the beam of electrons passing through.

7. **[15%] Tunneling junction** In quantum mechanics, the state of the electron with energy E moving in a potential U is described by a wavefunction $\varphi(z)$, which satisfied Schrodinger's equation. An electron with energy $E = 1$ eV is incident upon a rectangular barrier of potential energy $U = 2$ eV (See Fig. 1).

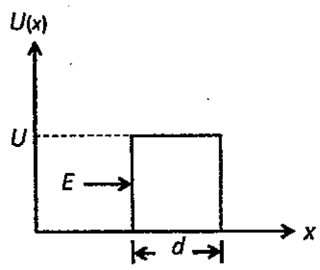

Fig. 1

(1) (10%) Let the width of the barrier d is 0.8 nm. Calculate the transmission probability **T** for an electron through the barrier.

(2) (5%) Figure 1 also represents the one-dimensional metal-vacuum-metal tunneling junction in STM measurements. By applying a bias voltage V, a sample state φ_n with energy level E_n lying between $E_F - eV$ and E_F has a chance to tunnel into the tip. Assuming that the bias (eV) is much smaller than the value of the work function (Φ), the probability **T** for an electron in the *n*th sample state to present at the tip surface can be described by $T \propto |\varphi_n|^2 e^{-2\kappa d}$, where $\kappa = \frac{\sqrt{2m\Phi}}{\hbar}$ is the decay constant of a sample state near the Fermi level in the barrier region. According to the above statement, try to give a brief explanation, why STM is a suitable tool to probe the electronic structure *at an atomic level.*

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