※ 注意：請於試卷內之「非選擇題作答區」依序作答，並應註明作答之大題及小題題號。

1. (12 points) For each of the following algorithm, what is the tightest asymptotic upper bound for its worst-case running time? Use the following table for your answers. You need not justify your answers.

A	B	C	D	E	F	G	H
$O(\lg n)$	$O(n)$	$O(n \lg n)$	$O(n^2)$	$O(n^2 \lg n)$	$O(n^{2.5})$	$O(n^{\lg 7})$	$O(n^3)$

   (a) Insertion sort for n numbers
   (b) Strassen's algorithm for multiplying two $n \times n$ matrices
   (c) Graham scan for n points
   (d) BUILD-HEAP for n numbers
   (e) Merge sort for n numbers
   (f) Floyd-Warshall algorithm for a graph of n vertices

2. (18 points) Give tight asymptotic bounds for the following recurrences. You need not justify your answers.

   (a) (6 points) $T(n) = \begin{cases} 3T(n/8) + \sqrt{n} & \text{if } n > 1 \\ 1 & \text{if } n = 1, \end{cases}$

   (b) (6 points) $T(n) = \begin{cases} T(\sqrt{n}) + 1 & \text{if } n > 2 \\ 1 & \text{if } n \leq 2, \end{cases}$

   (c) (6 points) $T(n) = \begin{cases} 8T(n/2) + \Theta(1) & \text{if } n^2 > M \\ M & \text{if } n^2 \leq M, \end{cases}$ where M is a variable independent from n.

3. (15 points) Various collision resolution strategies have been explored to tackle the problem of hash collisions. Given a hash table H with m slots that store n elements, please answer the following questions under the assumption of simple uniform hashing.

   (a) (5 points) What is the expected time complexity of searching in H if
      i. H resolves collisions by chaining and each chain is stored in an AVL tree.
      ii. another hash table H' is used to hash all the elements with the same hash value in H. Collisions in H' are resolved recursively. Assume that each H' also has m slots.

   (b) (5 points) Determine m (in terms of n) such that the expected number of pairwise collisions can be reduced to $O(1)$. Show your calculation.

   (c) (5 points) Suppose H maps elements to values from 0 to 6, i.e., m=7. Draw H that results from inserting 8, 22, 16, 28, 11 (in order) into an initially empty H by double hashing, where the primary hash function is defined as $h_1(k) = k \mod 7$ and the secondary hash function is defined as $h_2(k) = 1 + (k \mod 5)$.

4. (15 points) Different tree-based data structures such as min-heap, binary search tree (BST) and red-black tree (RB-tree) satisfy different properties.

   (a) (5 points) Give a recursive algorithm for determining whether a binary tree satisfies the BST properties. The input is the root of a binary tree. The algorithm returns TRUE if the binary tree is a BST and FALSE if it isn't. Your algorithm must use at most a constant amount of extra space and its running time must be $O(n)$, where n denotes the number of the nodes in the binary tree.

   (b) (5 points) Give a recursive algorithm for determining the number of different BSTs with n nodes, where n is the input.

   (c) (5 points) Draw a smallest AVL tree of height 3 (number of edges from the root to the leaves) and then label its nodes with either B (black) or R (red) to make it satisfy the RB-tree properties.

5. (12 points) Given a weighted directed graph $G_d = (V_d, E_d)$, as shown in Figure 1, where the weight of an edge $w_{G_d}(e)$ is given by the number next to the edge, please determine the following:

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(a) (6 points) Using at most 4 edges, what is the minimum cost path from vertex A to vertex H? Please put down the sequence of vertices in the path in the order of visiting. In the case that there are multiple such minimum cost paths, you only need to put down one.

(b) (6 points) Convert $G_d = (V_d, E_d)$ to a weighted undirected graph $G_u = (V_d, E_u)$ as follows. An edge $e = (i, j) \in E_u$ iff $\langle i, j\rangle \in E_d$ or $\langle j, i\rangle \in E_d$. $\forall e = (i, j) \in E_u$, $w_{G_u}(e) = w_{G_d}(\langle i, j\rangle) + w_{G_d}(\langle j, i\rangle)$ if both $\langle i, j\rangle \in E_d$ and $\langle j, i\rangle \in E_d$. Otherwise, $w_{G_u}(e) = w_{G_d}(\langle i, j\rangle)$ if $\langle i, j\rangle \in E_d$, and $w_{G_u}(e) = w_{G_d}(\langle j, i\rangle)$ if $\langle j, i\rangle \in E_d$. Please give the cost of G_u's minimum spanning tree(s).

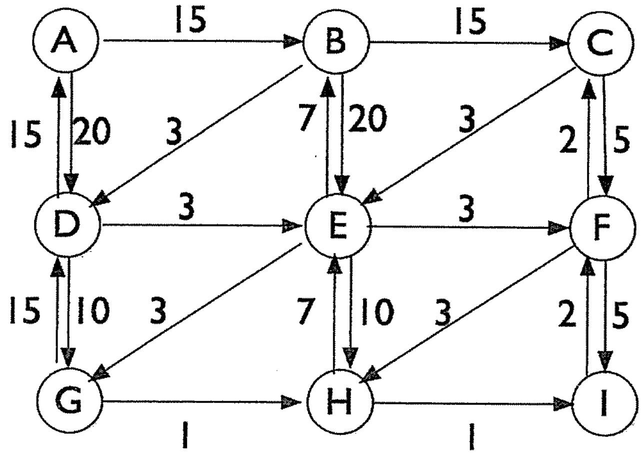

Figure 1: Directed graph G

6. (10 points) Consider an ordinary binary min-heap data structure H with n elements supporting the instructions INSERT and EXTRACT-MIN in $O(\log n)$ worst-case time. Give a potential function $\Phi(H)$ such that the amortized cost of INSERT is $O(\log n)$ and the amortized cost of EXTRACT-MIN is $O(1)$. The potential function should be zero when the heap is empty, and has positive value otherwise. Show that it works for both instructions.

7. (18 points) A *palindrome* is a string that reads the same from the left and from the right. For example, REDDER, I, ROTATOR, MOM, are palindromes. From a non-palindrome string, you can always remove an arbitrary number of characters from it to make it a palindrome. For example, from both ADAM and ADMA, you can remove 'M' to obtain a palindrome, ADA. In both cases, we say that ADA is the longest palindrome subsequence in the original strings, ADAM and ADMA.

We would like to derive a dynamic programming algorithm to determine the length of the longest palindrome subsequence in a string. You are given the string S of length n in the form of a character array $S = s[]$, where $s[i]$ is the i-th character in the string, $1 \le i \le n$.

(a) (12 points) As the first step, please complete the following equation to represent the length of the longest palindrome subsequence in the given string S by filling out the blanks. $L(i, j)$ represents the length of the longest palindrome subsequence in $s[i..j]$, a substring of S that starts from the i-th character and ends at the j-th character of S.

$L(i, j)$ is defined for $1 \le i \le n, 1 \le j \le n, j \ge i - 1$, and

$$L(i,j) = \begin{cases} \underline{\quad(1)\quad} & \text{, if } i = j + 1 \\ \underline{\quad(2)\quad} & \text{, if } i = j \\ \underline{\quad(3)\quad} & \text{, if } i < j \text{ and } s[i] == s[j] \\ \underline{\quad(4)\quad} & \text{, otherwise.} \end{cases}$$

(b) (6 points) Calculate the length of the longest palindrome sequence in the string CABDAACBADFA.

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