第一大題 1-5 選擇題考生請用 2B 鉛筆作答於答案卡，並先詳閱答案卡上之「畫記說明」。
Questions 1-5 are mixed with **single-choice** and **multiple-choice** questions.

1. (10%) Suppose a 32-pound weight stretches a spring 2 feet. If the weight is release from rest at the equilibrium position, find the equation of motion $x(t)$ if an impressed force $f(t)=\sin(t)$ acts on the system for $0 \le t < 2\pi$, and is then removed. Ignore any damping forces.

(A) $f(t)=\sin(t)-\sin(t)U(t-2\pi)$, (B) $x(t)=\frac{-1}{10}\sin 4t+\frac{1}{30}\sin t, 0 \le t < 2\pi$,

(C) $x(t)=\frac{-1}{60}\sin 4t+\frac{1}{15}\sin t, 0 \le t < 2\pi$, (D) $x(t)=\frac{-1}{30}\sin 4t+\frac{1}{5}\sin t, 0 \le t < 2\pi$,

2. (10%) A uniform 10-foot-long chain is coiled loosely on the ground. One end of the chain is pulled vertically upward by means of constant force of 2 pounds. The chain weights 1 pound per foot. Determine the height of the end above ground level, x(t), at time t.

(A) $x\frac{d^2x}{dt^2}+(\frac{dx}{dt})^2+16x=160$, (B) $x\frac{d^2x}{dt^2}+(\frac{dx}{dt})^2+32x=64$,

(C) $x\frac{d^2x}{dt^2}+(\frac{dx}{dt})^2+32x=160$, (D) $x=3-3(1-\frac{4}{3}t)^2$, (E) $x=\frac{15}{2}-\frac{15}{2}(1-\frac{4\sqrt{10}}{15}t)^2$

3. (10%) A semi-infinite plate coincides with the region defined by $0 \le x \le \pi, y \ge 0$. The right end is held at temperature $e^{-y}$, and the left end is held at temperature zero. The bottom of the plate is held at temperature $f(x)$. Find the steady-state temperature in the plate: $u(x,y)=\sum_{n=1}^{\infty}\frac{2}{\pi}A_n e^{-ny}+\frac{2}{\pi}\int_0^{\infty}B(\alpha)\sin(\alpha y)d\alpha$

(A) $A_n=\sin(nx)\int_0^{\pi}f(x)\sin(nx)dx$, (B) $A_n=\int_0^{\pi}f(x)\sin(nx)dx$,

(C) $B(\alpha)=\frac{\sinh(\alpha x)}{(1+\alpha^2)\sinh(\alpha\pi)}$, (D) $B(\alpha)=\frac{\alpha\sinh(\alpha x)}{(1+\alpha^2)\sinh(\alpha\pi)}$

4. (10%) Use the power series method to solve the DE $xy''+2y'-xy=0$, $y=c_1y_1+c_2y_2$

(A) $y_1=\sum_{n=0}^{\infty}\frac{1}{(2n+1)!}x^{2n+1}$, (B) $y_1=\sum_{n=0}^{\infty}\frac{1}{(2n+1)!}x^{2n}$, (C) $y_2=\sum_{n=0}^{\infty}\frac{1}{(2n)!}x^{2n}$,

(D) $y_2=\sum_{n=0}^{\infty}\frac{1}{(2n)!}x^{2n-1}$, (E) $y_1=\frac{\sinh(x)}{x}$

5. (10%) Solve the given equation $\int_0^t f(\tau)f(t-\tau)d\tau=6t^3$,

(A) $f(t)=6t$, (B) $f(t)=3\sqrt{2}t$, (C) $f(t)=\sqrt{6}t$, (D) $f(t)=-6t$, (E) $f(t)=-2t$

見背面

**第二大題考生應作答於『試卷』**

1. (20%) Label the following statements as being true or false. *(No explanation is needed. Each correct answer gets 2% and each wrong answer gets 0%)*:

   (a) A set $V$ is a vector space if $V$ satisfies the following properties: (i) $V$ has a zero vector; (ii) whenever $\mathbf{u}$ and $\mathbf{v}$ belong to $V$, then $\mathbf{u}+\mathbf{v}$ belongs to $V$; and (iii) whenever $\mathbf{v}$ belongs to $V$ and $c$ is a scalar, then $c\mathbf{v}$ belongs to $V$.

   (b) Let $B$ be an $m \times m$ invertible matrix and $A$ be an $m \times n$ matrix. Then $A$ and $BA$ have the same reduced row echelon form.

   (c) An $n \times n$ matrix $A$ is diagonalizable if and only if $BAB^T$ is diagonalizable.

   (d) Let $A$ be $n \times n$. Then rank $A =$ rank $A^2$

   (e) If $B$ is obtained from $A$ by applying a series of elementary row operations, then $A$ and $B$ have the same reduced row echelon form.

   (f) Let $\mathcal{S} = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$ be a linearly independent subset of $\mathcal{R}^n$ and $\mathbf{u}$ be a vector in $\mathcal{S}^\perp$. Then $\{\mathbf{u}, \mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_k\}$ is linearly independent.

   (g) Let $A$ be an $m \times n$ matrix. If $A\mathbf{u} = A\mathbf{v}$ implies $\mathbf{u} = \mathbf{v}$, then rank $A = n$.

   (h) If $\mathbf{v}$ is an eigenvector of $A^2$, then $\mathbf{v}$ is an eigenvector of $A$.

   (i) Let $A$ be $n \times n$. Then $\det(2A) = 2 \det A$.

   (j) If $\mathbf{v}$ is not a linear combination of $\{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_k\}$, then rank $[\mathbf{u}_1\ \mathbf{u}_2\ \ldots\ \mathbf{u}_k\ \mathbf{v}]$ $= 1+$ rank $[\mathbf{u}_1\ \mathbf{u}_2\ \ldots\ \mathbf{u}_k]$.

2. Let $\mathcal{B} = \{e^t,\ te^t,\ t^2e^t\}$, $V =$ Span $\mathcal{B}$, and $T$ be a linear operator on $V$ defined by $T(f) = f'(t)$.

   (a) (5%) Find $[T]_{\mathcal{B}}$, the matrix representation of $T$ with respect to $\mathcal{B}$.

   (b) (5%) Find the eigenvalues of $T$ and a basis for each eigenspace.

   (c) (5%) Is $T$ invertible? If it is, find $T^{-1}(c_1e^t + c_2te^t + c_3t^2e^t)$.

3. Let $V_1 =$ Span $\{\mathbf{v}_1, \mathbf{v}_2\}$ and $V_2 =$ Span $\{\mathbf{v}_3, \mathbf{v}_4\}$, where

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}.$$

   (a) (5%) Find an orthogonal basis for $V_1$.

   (b) (5%) Find a basis for Null $([\mathbf{v}_3\ \mathbf{v}_4]^T)$.

   (c) (5%) Let $W$ be the intersection of $V_1$ and $V_2$. Find a basis for $W$.