

1. Evaluate:

(a) $\lim_{x \rightarrow 0^+} \frac{\ln \cos 3x}{\ln \cos 2x}$, (5%)

(b) $\int_0^{\pi} \frac{x \sin x}{1 + \cos^2 x} dx$, (5%)

(c) $\int_0^1 \frac{1 - e^{-x^2}}{x^2} dx$, (10%)

2. Prove that if $0 < \kappa < 1$, (10%)

$$K(\kappa) = \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - \kappa^2 \sin^2 \theta}} = \frac{\pi}{2} \left\{ 1 + \left(\frac{1}{2}\right)^2 \kappa^2 + \left(\frac{1 \times 3}{2 \times 4}\right)^2 \kappa^4 + \left(\frac{1 \times 3 \times 5}{2 \times 4 \times 6}\right)^2 \kappa^6 + \dots \right\}.$$

3. Prove directly that a polynomial function $f: R^n \rightarrow R$ is uniformly continuous on the set $\{|x| \leq K\}$ for any fixed K . [First show it is bounded, i.e. $|f(x)| < K'$. Then consider $f(x+h) - f(x)$.] (10%)

4. Suppose that the sequence $\{a_n\}_{n=1}^{\infty}$ satisfies the following condition: (10%)

There is an $r, 0 < r < 1$, such that

$$|a_{n+1} - a_n| < br^n, \quad n = 1, 2, \dots,$$

where b is a positive constant. Show that this sequence converges.

5. (10%) Find the function whose tangent line has slope $x \ln \sqrt{x}$ for each value of $x > 0$ and whose graph passes through the point $(2, -5)$.

6. (10%) Find the volume of the solid generated when the region under the curve $y = \tan x$ over the region $0 \leq x \leq \pi/4$ is rotated about the x axis.

7. (10%) Find the least squares approximation $h(x) = a_0 + a_1x + a_2x^2$ for $f(x) = e^x, 0 \leq x \leq 1$. (If the answers are with decimal numbers, please round to the nearest hundredth.)

[Hint: Minimize the integral of the squared difference between $f(x)$ and $h(x)$.]

8. (10%) Determine the value of c for which the following series converges.

$$\sum_{k=3}^{\infty} \frac{1}{k(\ln k)^c}$$

9. (10%) A company's demand equation is $x = \sqrt{2000 - p^2}$, where p is the price in dollars and x is the quantity of demand. Find dp/dx and $d \ln(p)/d \ln(x)$ when $p = 40$ and interpret your answer.

試題隨卷繳回