

共 9 大題。總分 100 分。

1. 【15分】 If $A=\{x, a, b, c, d\}$, how many closed binary operations on A have x as the identity?
2. 【15分】 Use a C-like language to write a *recursive algorithm* to generate all the staircase paths from (x_0, y_0) to (x_1, y_1) one and only once.
3. 【10分】 Determine the number of staircase paths from $(0, 8, 1)$ to $(7, 4, 9)$.
4. 【10分】 Analyze the number of iterations in the following loops.

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for(i=1; i<=80; i+=4) for(j=1; j<=i; j+=2) for(k=1; k<=j; k++) { printf("\n"); }
```
5. 【10分】 In how many ways can we distribute 8 red balls and 9 blue balls among 6 children so that each child receives at least one blue ball?
6. 【10分】 For finite sets A and B with $|A|=m$ and $|B|=n$, determine the number of functions and onto functions from A to B .
7. 【10分】 Prove: generalized DeMorgan's law.
8. 【10分】 Prove: $A \rightarrow B \vee C = (A \wedge \neg B \rightarrow C)$ and $A \vee B \rightarrow C = (A \rightarrow C) \wedge (B \rightarrow C)$.
9. 【10分】 Prove: for all $n \geq 14$, n can be written as a sum of 3's and/or 8's.