1. (25%) Consider a point in a structural member that is subjected to plane stress. The stress state of this point is described by $\sigma_{xx} = 2a_1b_1$, $\sigma_{yy} = 2a_2b_2$, $\sigma_{xy} = a_1b_2 + a_2b_1$.

   (a). (10%) Let σ_{nn} (σ_{tt}) be the normal stress acting on the plane oriented at an angle θ $(\theta + \pi/2)$ with respect to the reference axis x. Suppose that $\sigma_{xx} + \sigma_{yy} = 0$. Show that there exists an angle θ such that $\sigma_{nn} = 0$, $\sigma_{tt} = 0$.

   Next, suppose further $\sigma_{xx} + \sigma_{yy} = 0$, $a_1^2 + a_2^2 = b_1^2 + b_2^2 = k^2$, and $k > 0, \frac{a_1}{a_2} = \sqrt{3}$.

   Assume $a_1 > 0$, $a_2 > 0$, $b_1 < 0$, $b_2 > 0$.

   (b). (10%) Find out the principal stresses and principal angle θ_p.

   (c). (5%) Find the maximum in-plane shearing stress and θ_τ.

2. (10%) Fig. 1 shows a loaded structure whose members can sustain only axial forces. The load P acts on the point B. The cross-sectional areas are A for both members AB and BC. The length of the member AB is l. Further both members have a modulus of elasticity E. Assume $\sin\theta = \frac{3}{5}$ and $\frac{Pl}{EA} = \delta^*$. Determine the horizontal u_1 and vertical u_2 components of the displacement of point B in terms of δ^*.

3. (15%) The assembly shown in Fig. 2 consists of a steel rod A (Young's modulus= E_A, area=A_A, length=L_A, coefficient of thermal expansion=α_A), a rigid bearing plate that is securely fastened to bar A, and a bronze bar B (Young's modulus= E_B, area=A_B, length=L_B, coefficient of thermal expansion=α_B). A clearance of δ_C exists between the bearing plate and bar B before the assembly is loaded. Suppose the load P is applied to the bearing plate and the temperature of the assembly is slowly raised. Assume $\frac{PL_A}{E_A A_A} = \frac{1}{2}\delta_C$. Find the temperature increase ΔT such that the stress in the steel rod A **begins decreasing**. Note that ΔT has to be expressed in terms of δ_C, L_A, L_B, α_A and α_B.

4. (25%) A prismatic tube AB of circular cross section (diameter ***d***, thickness ***t***, and length ***L***) is fixed at the bottom end A and free at the top end B (see Figure 3). The bar is loaded by a distributed torque of constant intensity ***T*** per unit length.
   (a). (10%) What is the maximum shear stress in the bar?
   (b). (15%) What is the angle of twist at the top end B?

5. (25%) The beam shown in Figure 4 has length of $3L$ and constant flexural rigidity EI. It is loaded uniformly by w (N/m).
   (a). (10%) Determine the reactions at both ends.
   (b). (10%) Determine the deflection at the middle point of the beam.
   (c). (5%) Determine the slope at the middle point of the beam.

見背面

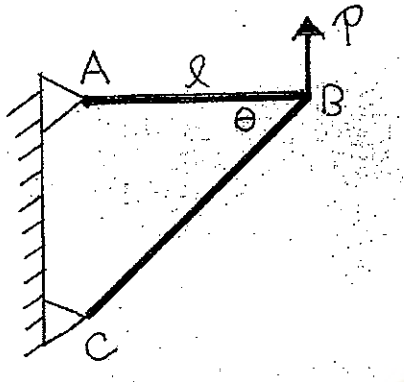

Fig. 1

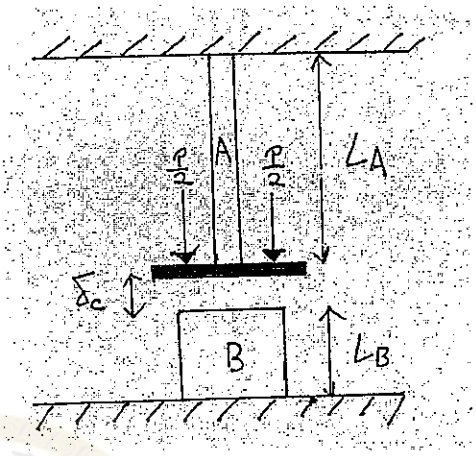

Fig. 2

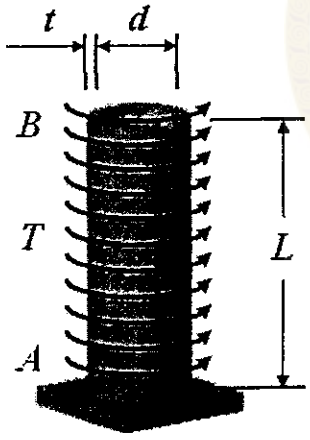

Fig. 3

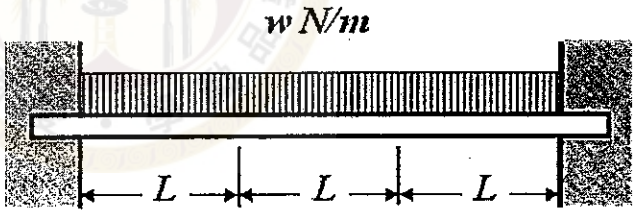

Fig. 4

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