1. Find the solutions:

(1) $\dfrac{d^2x}{dt^2}+4x=2\cos 3t+3\sin 3t$; $x=3$ *and* $\dfrac{dx}{dt}=2$ for $t=0$ (10%)

(2) $\int_0^{\pi}\dfrac{d\theta}{a+b\cos\theta}$; $a>b>0$ (10%)

2. Solve the following system of differential equations

$$\frac{d^2x}{dt^2}+2x-y=0$$

$$\frac{d^2y}{dt^2}+2y-x=0$$

with initial conditions: $x(0)=C_1$ $x'(0)=C_2$, $y(0)=C_3$, $y'(0)=C_4$ (20%)

3. $[A]=\begin{bmatrix}1 & -1\\ 4 & 1\end{bmatrix}$

(1) Find all eigenvalues and associated eigenvectors. (10%)

(2) Find a matrix [C] and a diagonal matrix [B] such that $[A]=[C]^{-1}[B][C]$ (10%)

4. (1) Given the vectors $\vec{u}=\vec{i}-2\vec{j}+\vec{k}$, $\vec{v}=3\vec{i}+\vec{k}$, $\vec{w}=\vec{j}+\vec{k}$

Evaluate $(\vec{u}+\vec{v})\cdot(\vec{v}+\vec{w})\times\vec{w}$ (10%)

(2) Find the area of the triangle whose vertices are (0,0,0), (0,1,0) and (1,1,1) by vector means. (10%)

5. In this problem we attempt to obtain the Fourier cosine coefficient of $\cosh x$

$$\cosh x = A_0+\sum_{n=1}^{\infty}A_n\cos\frac{n\pi x}{L} \qquad (1)$$

Differentiating yields

$$\sinh x=-\sum_{n=1}^{\infty}\frac{n\pi}{L}A_n\sin\frac{n\pi x}{L}$$

The Fourier sine series of $\sinh x$. Differentiating again yields

$$\cosh x=-\sum_{n=1}^{\infty}\left(\frac{n\pi}{L}\right)^2A_n\cos\frac{n\pi x}{L} \qquad (2)$$

Since equations (1) and (2) give Fourier cosine series of $\cosh x$, they must be identical. Thus, $A_0=0$, $A_n=0$. (obviously wrong.)

Please find mistakes and correct them. By correcting the mistakes you should be able to obtain A_0 without using the typical technique, that is, $A_0=\frac{1}{L}\int_0^L\cosh x\,dx$.

$\left(\text{Hint}: f(x)\approx\sum_{n=1}^{\infty}B_n\sin\frac{n\pi x}{L} \quad \text{and} \quad f'(x)\approx A_0+\sum_{n=1}^{\infty}A_n\cos\frac{n\pi x}{L}\right)$ (20%)

試題隨卷繳回