※ 注意：請於試卷內之「非選擇題作答區」標明題號依序作答。

1. (20 points) An infinite planar current sheet, $\bar{J}_S$, is placed at the interface (at $z=0$) of two perfect dielectric materials, where $\bar{J}_S$ is a time-varying surface current density by
$\bar{J}_S = 0.2\cos(8\pi\times10^8 t)\hat{x}\ (A/m)$.

The values of permittivity in these two regions are given by $\varepsilon = 4\varepsilon_0$ and $\varepsilon = 9\varepsilon_0$ for $z<0$ and $z>0$, respectively. (Both cases assume $\mu=\mu_0$)

(1) (4 points) Find the intrinsic impedances, η_1 and η_2, in these two regions of $z<0$ and $z>0$, respectively. Also find the corresponding phase constants, β_1 and β_2. (Note that the speed of light is 3×10^8 m/s)

(2) (8 points) Find the electric field $\bar{E}(z,t)$ and the magnetic field $\bar{H}(z,t)$ in both regions $z>0$ and $z<0$.

(3) (4points) Specify the boundary conditions for $\bar{E}(z,t)$ and $\bar{H}(z,t)$, respectively at $z=0$ when the field points in the two regions go close to the interface at $z=0$.

(4) (4 points) Please show if the time-average power densities in these two regions are equal?

2. (15 points) Consider two parallel infinite plane sheets held a distance d apart (the material inside is air). The surface charge density is ρ_{S0} at $z=0$, and is $-\rho_{S0}$ at $z=d$.

(1) (6 points) Find the electric field everywhere. Calculate the voltage between the two plane sheets. Find the capacitance per area.

(2) (6 points) Now the air is replaced by a dielectric material, $\varepsilon = 2.25\varepsilon_0$. Repeat the computations in (1).

(3) (3 points) Find the polarization current density, $\bar{J}_p$.

3. (15 points) Consider an electric field propagating radially outward in free space, which is expressed in **spherical** coordinates by

$$\bar{E} = E_0 \frac{\sin\theta}{r}\cos\left[\omega\left(t - r\sqrt{\mu_0\varepsilon_0}\right)\right]\hat{a}_\theta \qquad \text{(V/m)},$$

where (r,θ,ϕ) is the variables of spherical coordinate system with $(\hat{a}_r,\hat{a}_\theta,\hat{a}_\phi)$ being the unit vectors of their axes. We assume that $\bar{E}$, $\bar{H}$ and their propagation direction follow the rule of plane wave.

(a) (5 points) Please find the propagation direction, and afterward determine the magnetic field, $\bar{H}$.

(b) (5 points) Find the instantaneous power density ($\bar{P} = \bar{E}\times\bar{H}$). Specify the direction of power propagation.

(c) (5 points) Integrate the power density in (b) through a **spherical** surface with a radius r to find the power ($\oint \bar{P}\cdot d\bar{s}$). Also calculate the **time average** power. (Note: $d\bar{s} = (r^2\sin\theta d\theta d\phi)\hat{a}_r$)

4. (15 points) Consider a transmission line with characteristic impedance Z_0, propagation constant β, and length l. It is connected to a short-circuited load at one end.

(a) (5 points) Show that the input impedance looking into the transmission line (from the other end) can be expressed as $Z_{in} = jZ_0\tan\beta l$

(b) (5 points) Assume that the short-circuited load is replaced by an arbitrary load Z_L, as shown in

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Figure 1, please show that the input impedance looking into the transmission line can be expressed as $Z_{in} = Z_0 \dfrac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}$

(c) (5 points) Let $Z(d)$ be the line impedance at location d looking towards the load. Show that $Z(d)\cdot Z\left(d+\dfrac{\lambda}{4}\right)=Z_0^2$. (Hint: use $Z(d)=Z_0\dfrac{1+\Gamma(d)}{1-\Gamma(d)}$, where $\Gamma(d)$ is the reflection coefficient at location d)

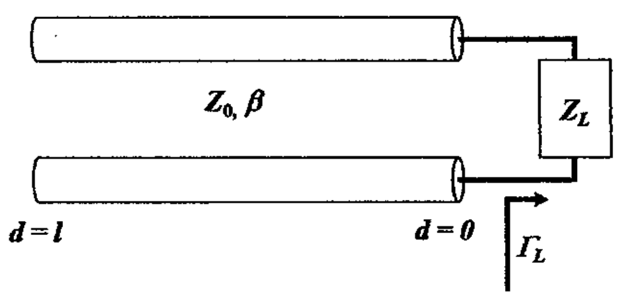

Figure 1.

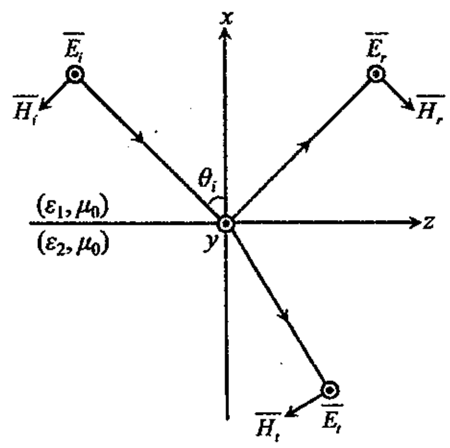

Figure 2

5. (15 points) Figure 2 shows a plane wave of TE polarization incident upon a planar interface at $x=0$, the electric field is $\bar{E}_i(\bar{r})=\hat{y}E_0 e^{jk_{1x}x-jk_z z}$, the reflected electric field is $\bar{E}_r(\bar{r})=\hat{y}RE_0 e^{-jk_{1x}x-jk_z z}$, and the transmitted electric field is $\bar{E}_t(\bar{r})=\hat{y}TE_0 e^{jk_{2x}x-jk_z z}$, where R is the reflection coefficient and T is the transmission coefficient.

(a) (5 points) Derive the incident, reflected and transmitted magnetic fields by using $\bar{H}(\bar{r})=\dfrac{-1}{j\omega\mu}\nabla\times\bar{E}(\bar{r})$.

(b) (5 points) Impose the boundary condition that the tangential electric and magnetic fields are continuous at $x=0$ to solve R and T.

(c) (5 points) If $\varepsilon_1>\varepsilon_2$, derive the formula of the critical angle $\theta_i=\theta_c$ at which total reflection occurs.

6. (20 points) The vector potential $\bar{A}(\bar{r})$ induced by a Hertzian dipole $\hat{z}I\ell\delta(\bar{r})$ at the origin can be represented in the spherical coordinate as $\bar{A}(\bar{r})=\hat{z}\dfrac{\mu I\ell}{4\pi r}e^{-jkr}$.

(a) (5 points) Derive the magnetic field in the spherical coordinate by using $\bar{H}(\bar{r})=\dfrac{1}{\mu}\nabla\times\bar{A}(\bar{r})$.

(b) (5 points) Derive the electric field in the spherical coordinate by using in the spherical coordinate by using $\bar{E}(\bar{r})=\dfrac{1}{j\omega\varepsilon}\nabla\times\bar{H}(\bar{r})$.

(c) (5 points) In the far field ($kr\gg 1$), write down the dominant term in the expressions of $\bar{H}(\bar{r})$ in (a) and $\bar{E}(\bar{r})$ in (b).

(d) (5 points) Derive the time-average power density by using the far-field expressions in (c).

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