*各題答案應作答於答案卡上，否則不予計分。
*每題有一個或一個以上正確選項，完整答對(無任何選項答錯)，該題得滿分。
*每題未作答或答錯(應選而未選或不應選而選) ，該題以 0 分計算。

1. (5%) Which are the possible solutions of the differential equation $\frac{dy}{dx}=y^2-9$?

   (A) $y=3$
   (B) $y=-3$
   (C) $y=3\frac{1+e^{6x}}{1-e^{6x}}$
   (D) $y=3\frac{1+e^{-6x}}{1-e^{-6x}}$
   (E) $y=3\frac{1-e^{-6x}}{1+e^{-6x}}$

2. (5%) Which are the possible solutions of the differential equation $\frac{dy}{dx}+y=e^{-2x}$?

   (A) $y=e^{-x}$
   (B) $y=e^{-2x}$
   (C) $y=e^{-x}-e^{-2x}$
   (D) $y=-e^{-x}-e^{-2x}$
   (E) $y=e^{-x}+e^{-2x}$

3. (5%) Which statements are correct for the equation $(2y^2+3x)dx+2xydy=0$?

   (A) The differential equation is exact.
   (B) It is possible to find an integrating factor so that the equation becomes exact.
   (C) $x=-y^2$ is one possible solution.
   (D) $x^2y^2+x^3=1$ is one possible solution.
   (E) None of the above

4. (5%) Consider the equation $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+2y=2x-2$ with the boundary conditions $y(0)=0, y(\pi)=\pi$. Which statements are correct?

   (A) The particular solution is $y_p=-x$.
   (B) The general solution is $y=e^x\sin x+x$.
   (C) There is a unique solution to this boundary-value problem.
   (D) This differential equation is homogeneous.
   (E) None of the above.

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5. (5%) Which are the possible solutions of the differential equation $y^2\dfrac{d^2y}{dx^2}=\dfrac{dy}{dx}$?

(A) $y+\ln|y-1|=x$

(B) $2y+\ln|y-1|=4x$

(C) $2y+\ln|2y-1|=4x+4$

(D) $y^2=2x$

(E) $y^2=-2x+1$

6. (5%) Assume $y=\sum_{n=0}^{\infty}c_n x^n$ and solve $\dfrac{d^2y}{dx^2}-xy=0$. Which statements are always true?

(A) $c_0=0$

(B) $c_1=0$

(C) $c_2=0$

(D) $c_n=(n+2)(n+1)c_{n+3}, n=0,1,2,\cdots$

(E) $c_n=(n+3)(n+2)c_{n+3}, n=0,1,2,\cdots$

7. (5%) Consider the equation $\dfrac{dy}{dt}=1-\sin t-\int_0^t y(\tau)d\tau$ with the initial condition $y(0)=1$. Which statements about the function $y(t)$ and its Laplace transform $Y(s)$ are correct?

(A) $Y(s)=\dfrac{s+1}{s^2+1}-\dfrac{s}{(s^2+1)^2}$.

(B) $Y(s)=\dfrac{1}{s^2+1}-\dfrac{s}{(s^2+1)^2}$.

(C) $y(t)=\sin t-\dfrac{1}{2}t\sin t$

(D) $y(\pi)=0$

(E) $y(\pi)=-1$

8. (5%) Which are the possible solutions of the given system?

$$\begin{cases}\dfrac{dx}{dt}=3x+2y+2e^{-t}\\ \dfrac{dy}{dt}=-2x-y+e^{-t}\end{cases}$$

(A) $\begin{bmatrix}x\\y\end{bmatrix}=\begin{bmatrix}1/2\\2\end{bmatrix}e^{-t}$

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(B) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1/2 \\ -2 \end{bmatrix} e^{-t}$

(C) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} e^{t} + \begin{bmatrix} 1/2 \\ 2 \end{bmatrix} e^{-t}$

(D) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} te^{t} + \begin{bmatrix} 0 \\ 1/2 \end{bmatrix} e^{t} + \begin{bmatrix} 1/2 \\ -2 \end{bmatrix} e^{-t}$

(E) $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} te^{t} + \begin{bmatrix} 1 \\ -1/2 \end{bmatrix} e^{t} + \begin{bmatrix} 1/2 \\ -2 \end{bmatrix} e^{-t}$

9. (5%) Consider the wave equation $a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}, -\infty < x < \infty, t > 0$ with initial conditions $u(x,0) = f(x), \left.\frac{\partial u}{\partial t}\right|_{t=0} = 0$.

Which are the possible solutions?

(A) $u(x,t) = f(x-at)$

(B) $u(x,t) = f(x+at)$

(C) $u(x,t) = \frac{1}{2} f(x-at) + \frac{1}{2} f(x+at)$

(D) $u(x,t) = \frac{1}{2} f(x-at) - \frac{1}{2} f(x+at)$

(E) None of the above

10. (5%) Evaluate the Fourier transform $\int_{-\infty}^{\infty} e^{-\frac{t^2}{2\tau^2}} e^{i\omega_0 t} e^{i\omega t} dt$.

(A) $\sqrt{\pi}\tau e^{-(\omega-\omega_0)^2 \tau^2}$

(B) $2\sqrt{\pi}\tau e^{-(\omega-\omega_0)^2 \tau^2}$

(C) $\sqrt{2\pi}\tau e^{-(\omega-\omega_0)^2 \tau^2}$

(D) $2\sqrt{\pi}\tau e^{-(\omega-\omega_0)^2 \tau^2/2}$

(E) $\sqrt{2\pi}\tau e^{-(\omega-\omega_0)^2 \tau^2/2}$

11. (5%) Find an equation involving g, h, and k that makes this augmented matrix correspond to a consistent system:

$$\begin{bmatrix} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{bmatrix}$$

(A) 2k+g+h = 0

(B) k+2g+3h = 0

(C) k+2g+h = 0

(D) 3k+g+2h = 0

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(E) $4k+h+2h = 0$

12. (5%) Let $a_1=\begin{bmatrix}1\\4\\-2\end{bmatrix}$, $a_2=\begin{bmatrix}-2\\-3\\7\end{bmatrix}$, and $b=\begin{bmatrix}4\\1\\h\end{bmatrix}$. For what value(s) of h is b in the plane spanned by a_1 and a_2?

(A) 0
(B) -10
(C) 16
(D) -17
(E) 20

13. (5%) Determine the rank of the matrix in the following.

$$\begin{bmatrix}2 & 5 & -3 & -4 & 8\\4 & 7 & -4 & -3 & 9\\6 & 9 & -5 & 2 & 4\\0 & -9 & 6 & 5 & -6\end{bmatrix}$$

(A) 0
(B) 1
(C) 2
(D) 3
(E) 4

14. (5%) Matrix A can be written in the form A=LU, where L is m x m lower triangular matrix 1's on the diagonal and U is an m x n echelon form of A. Such a factorization is called an LU factorization. Please find the L matrix for LU factorization of the following matrix A.

$$A=\begin{bmatrix}3 & -6 & 3\\6 & -7 & 2\\-1 & 7 & 0\end{bmatrix}$$

(A) $\begin{bmatrix}1 & 0 & 0\\2 & 1 & 0\\\frac{-1}{3} & 1 & 1\end{bmatrix}$

(B) $\begin{bmatrix}1 & 0 & 0\\1 & 1 & 0\\\frac{-2}{3} & 2 & 1\end{bmatrix}$

(C) $\begin{bmatrix}1 & 0 & 0\\1 & 1 & 0\\\frac{-2}{5} & 2 & 1\end{bmatrix}$

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(D) $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ \frac{2}{5} & 1 & 1 \end{bmatrix}$

(E) $\begin{bmatrix} 1 & 0 & 0 \\ \frac{2}{5} & 1 & 0 \\ 2 & 1 & 1 \end{bmatrix}$

15. (5%) Find the determinant for the following matrix $\begin{bmatrix} 1 & a & b+c \\ 1 & b & a+c \\ 1 & c & a+b \end{bmatrix}$.

(A) -5
(B) 5
(C) 1
(D) -1
(E) 0

16. (5%) Find the dimension of the subspace $H = \left\{ \begin{bmatrix} a - 3b + 6c \\ 5a + 4d \\ b - 2c - d \\ 5d \end{bmatrix} : a, b, c, d \text{ in } \mathbb{R} \right\}$

(A) 2
(B) 3
(C) 4
(D) 1
(E) 0

17. (5%) Let $b_1 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$, $b_2 = \begin{bmatrix} -2 \\ 4 \end{bmatrix}$, $c_1 = \begin{bmatrix} -7 \\ 9 \end{bmatrix}$, $c_2 = \begin{bmatrix} -5 \\ 7 \end{bmatrix}$, and consider the bases for $\mathbb{R}^2$ given by $\boldsymbol{B} = \{b_1, b_2\}$ and $\boldsymbol{C} = \{c_1, c_2\}$.

Find the change-of-coordinates matrix from $\boldsymbol{C}$ to $\boldsymbol{B}$.

(A) $\begin{bmatrix} 5 & 3 \\ 6 & 4 \end{bmatrix}$

(B) $\begin{bmatrix} 5 & 3 \\ 2 & 6 \end{bmatrix}$

(C) $\begin{bmatrix} 2 & 7 \\ 1 & 4 \end{bmatrix}$

(D) $\begin{bmatrix} 5 & 3 \\ 7 & 4 \end{bmatrix}$

(E) $\begin{bmatrix} 1 & 3 \\ 6 & 4 \end{bmatrix}$

18. (5%) Which of the following statement is True?
    (A) If A is a 3 x 3 matrix, then det 5A=5 det A.
    (B) If $A^3=0$, then det A =0.
    (C) If **u** and **v** are in $\mathbb{R}^2$ and det[**u v**]=10, then the area of the triangle in the plane with verices a at 0, **u**, **v** is 10.
    (D) If A is n x n and det A=2, then det $A^3=6$.
    (E) Any system of n linear equations in n variables can be solved by Cramer's rule.

19. (5%) Find the singular values of the following matrix $\begin{bmatrix} \sqrt{6} & 1 \\ 0 & \sqrt{6} \end{bmatrix}$.
    (A) 0
    (B) 1
    (C) 2
    (D) 3
    (E) 4

20. (5%) Let A be n x n matrix. Choose the following statements equivalent to the statement that A is an invertible matrix.
    (A) Nul A= $\mathbb{R}^n$
    (B) Row A = $\mathbb{R}^n$
    (C) A has n nonzero singular values.
    (D) Dimension of Col A= 0 ,
    (E) The columns of A form a linearly independent set.

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