(1) (10 points) A rectangle with corners at $(-x, 0), (x, 0), (x, y), (-x, y)$ is inscribed in a half circle $x^2 + y^2 = 1$ where $y \geq 0$. Note that rectangle is in the upper half plane. Assume we move x on the half circle, $x^2 + y^2 = 1$, as $x(t) = t^2$ and t goes from 0 to 1.

(a) (7 points) Find the rate of change of $y(t)$.

(b) (3 points) Find the rate of change of the area $A(t) = 2x(t)y(t)$ of the rectangle.

(2) (10 points) (a) (5 points) Analyze the local extrema of the function

$$f(x) = \frac{x}{1 + x^2}$$

on the real axes using the second derivative test.
(b) (5 points) Are there any global extrema?

(3) (10 points) Suppose that $x^2y + xz^2 = 5$, and let $w = x^3y$. Express $\left(\frac{\partial w}{\partial z}\right)_y$ as a function of x, y, and z, and evaluate it numerically when $(x, y, z) = (1, 1, 2)$.

(4) (15 points) Consider the region R in the first quadrant bounded by the curves $y = x^2$, $y = x^2/5$, $xy = 2$, and $xy = 4$.

(a) (7 points) Compute $dxdy$ in terms of $dudv$ if $u = x^2/y$ and $v = xy$.

(b) (8 points) Find a double integral for the area of R in uv coordinates and evaluate it.

(5) (15 points) Find the Lagrange multiplier equations for the point of the surface

$$x^4 + y^4 + z^4 + xy + yz + zx = 6$$

at which x is largest. (Do not solve.) Explain clearly how you reach your conclusion.

(6) (10 points) Prove or disprove the following statement:

$$\frac{x}{1 + x^2} < \tan^{-1}(x) < x \quad \text{for all } x > 0.$$

(7) (15 points) (a) (3 points) Find the Taylor series of $\ln(1 + x)$ centered at $a = 0$.

(b) (5 points) Determine the radius of convergence of this Taylor series.

(c) (2 points) Use the first two non-zero terms of the power series you found in (a) to approximate $\ln(1.5)$.

(d) (5 points) Give an upper bound on the error in your approximation in (c) using Taylors inequality.

(8) (15 points) The point $x = \cos t$, $y = 5 + \sin t$ travels on a circle with center at $(0, 5)$. Revolving that circle around the x axis produces a doughnut. Find its surface area.

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